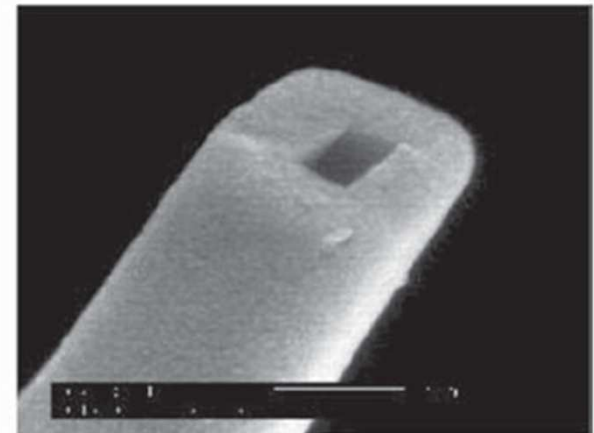
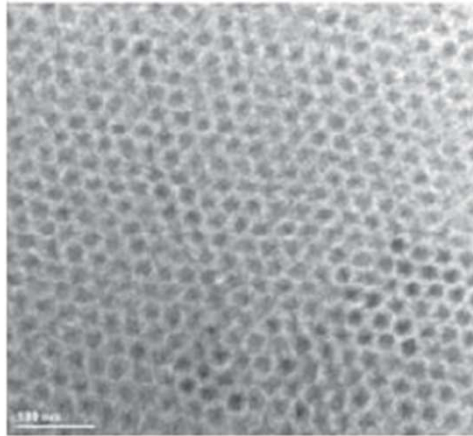
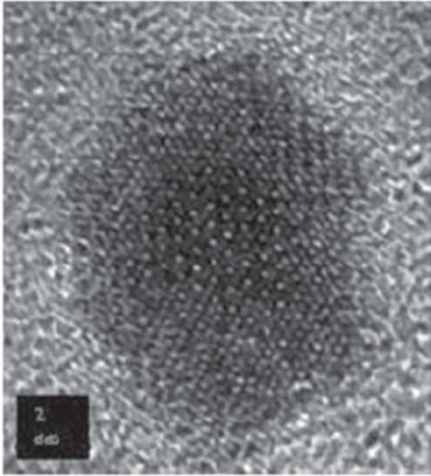
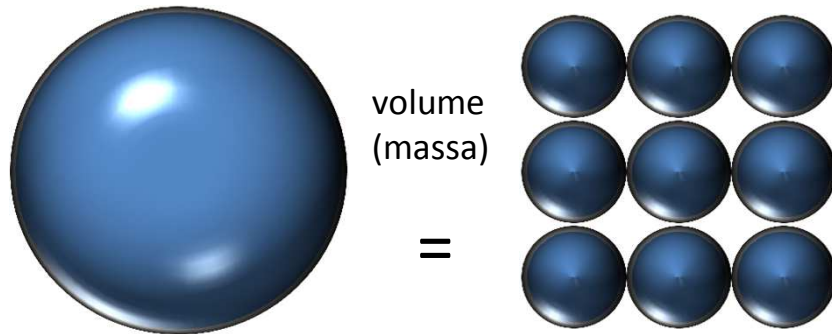


## NANOSTRUTTURE



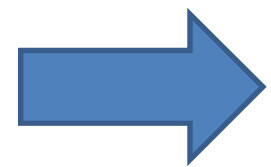
Nanoscale Science and Technology  
R. Kelsall, I. Hamley, M. Geoghegan  
Wiley (H-E4-16)



volume  
(massa)

=

$$\frac{\text{Superficie}}{\text{Volume}} \propto \frac{1}{r}$$



**Superficie  
specifica**  
(superficie/massa)

$$4\pi r^2 / (4/3\pi r^3 \rho) = 3/r\rho$$

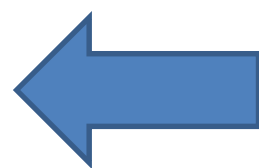
$$\rho = 2 \text{ g/cm}^3$$

$$\text{diam} = 4 \text{ nm}$$

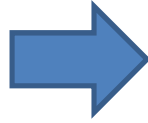
$$4\pi r^2 / (4/3\pi r^3) = 3/r$$



**Superficie specifica ~ 750 m<sup>2</sup>/g**



Nanoparticelle

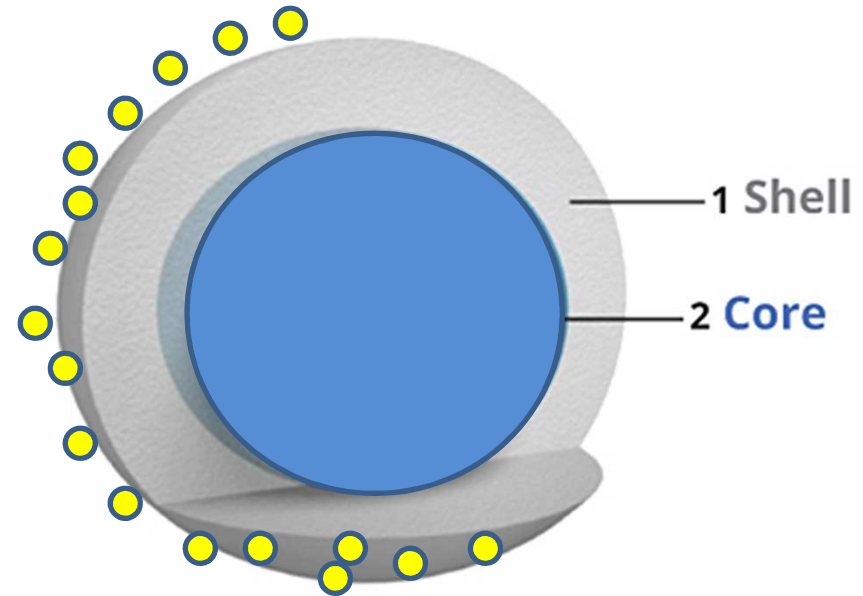


Grandi quantità di atomi di superficie

Differenze di proprietà rispetto agli atomi di bulk

- Energie di legame
- Strutture elettroniche

Fenomeni di adsorbimento



Distanza atomica =  $a$

$$N_{surf} = 4\pi r^2/a^2$$

$$N = (4/3\pi r^3)/a^3$$

$$N_{surf}/N \cong 3a/r$$

Per  $r = 6a$  ~50% atomi di superficie

Per cristalli  $a \sim 0.3$  nm

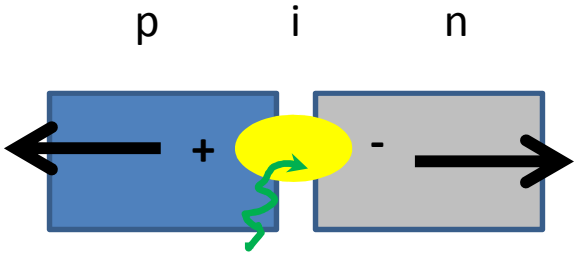
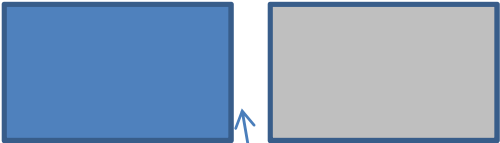
$6a \sim 2$  nm

Effetti energia superficie

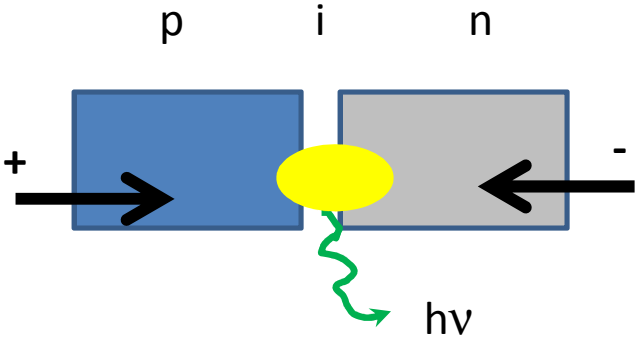
Nanostrutture in semiconduttori

p-doped                      n-doped

p                      i                      n

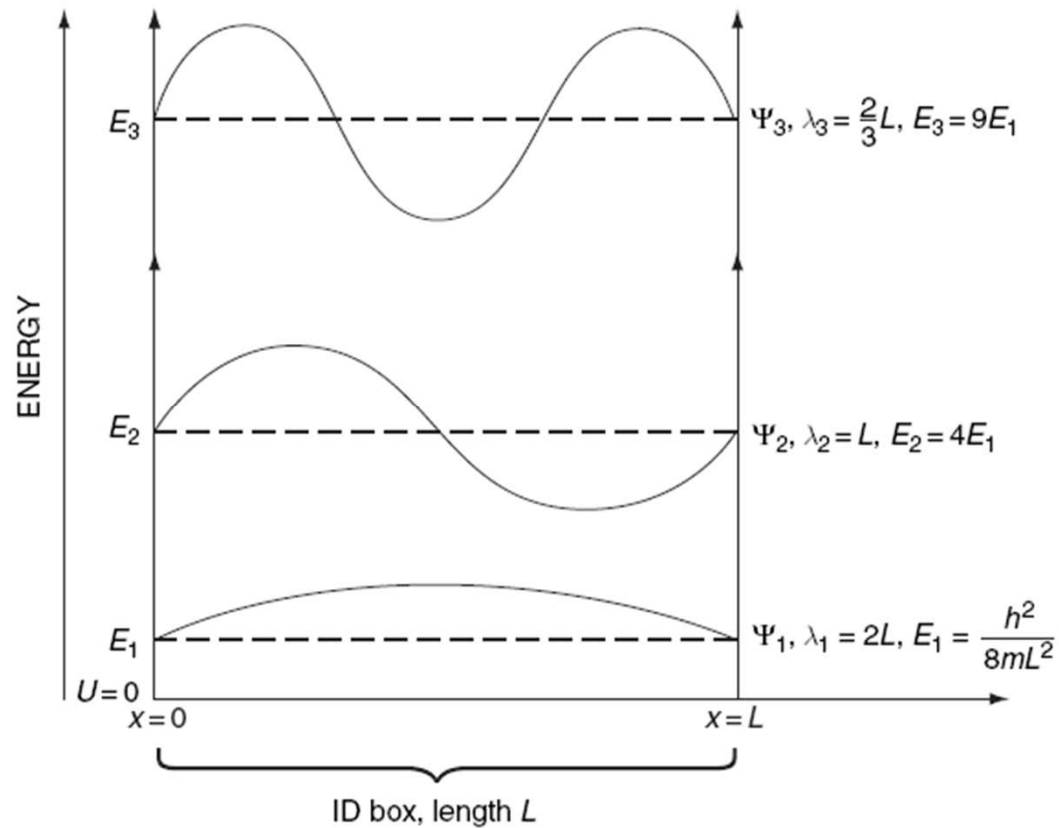


Generazione Coppie elettrone-lacuna  
Photon detector



ricombinazione Coppie elettrone-lacuna  
Photon emitter

## Elettrone in una scatola



## Soluzioni eq. Schrodinger

$$V = 0 \quad 0 < x < L$$

$$V = \infty \quad x < 0 \quad x > L$$

Le soluzioni devono avere

$$\lambda_n = 2L/n$$

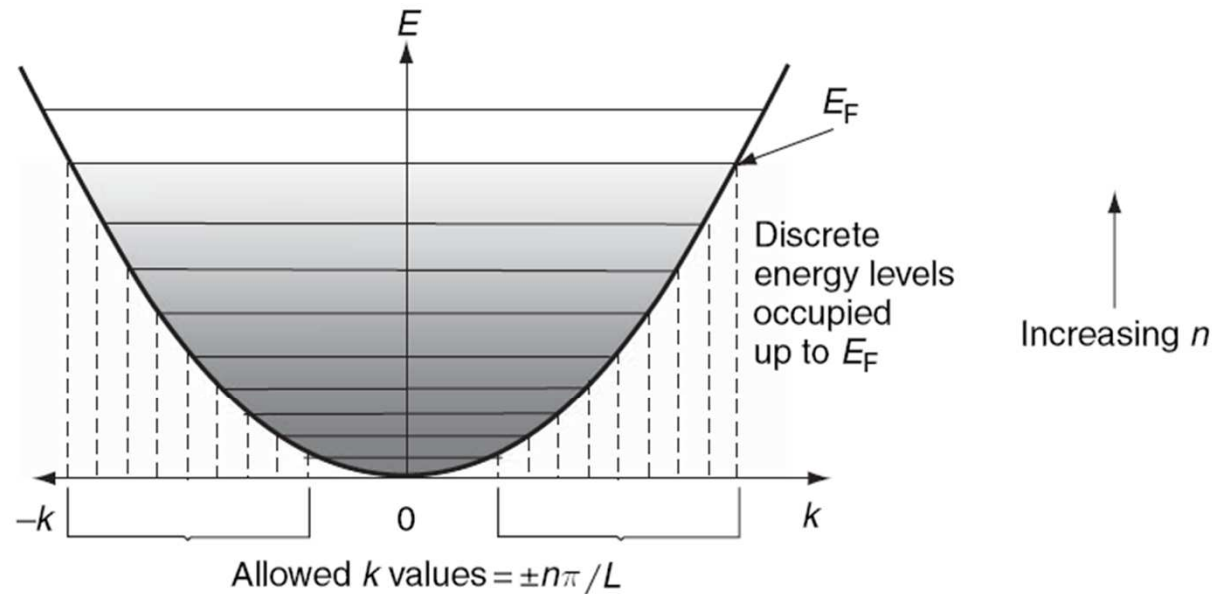
$$n = 1, 2, 3 \dots$$

$$k_n = 2\pi/\lambda \quad k_n = n\pi/L$$

$$\psi_n = (2/L)^{1/2} \sin(n\pi x/L)$$

$$P^2/2m \quad \rightarrow \quad E_n = \frac{n^2 h^2}{8mL^2}$$

Separazione livelli dipendente da  $L$



**Figure 1.7** Schematic version of the parabolic relationship between the allowed electron wave vectors and their energy for electrons confined to a one-dimensional potential well. Shaded energy regions represent those occupied with electrons

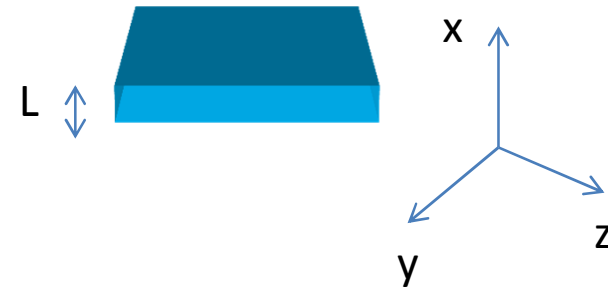
$$E_n = \frac{n^2 h^2}{8mL^2} \quad k_n = n\pi/L$$

Dipendenza parabolica dell'energia da  $k$   
*Discretizzazione valori di  $K$  e di  $E$*

## CONFINAMENTO QUANTICO : QUANTUM WELL - 2D

Spessore  $L \ll$  dimensioni  $y, z$

Il moto e' vincolato lungo  $x$   
 Buca quantica infinita



time-independent Schrödinger equation

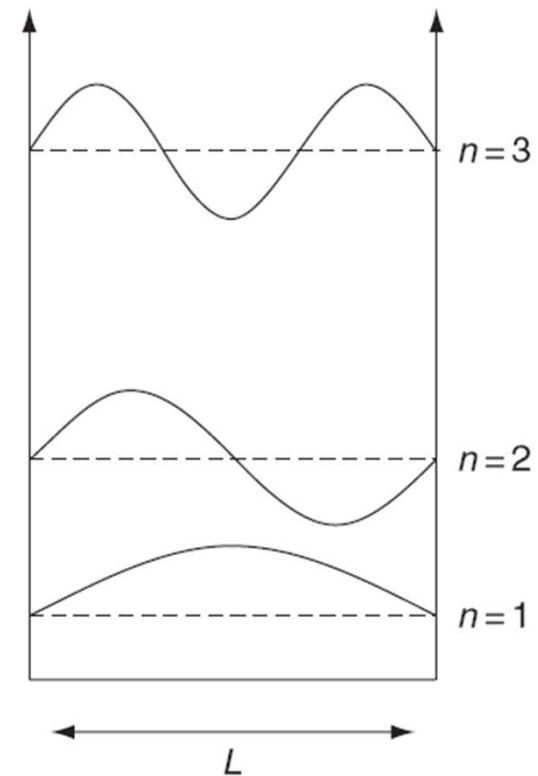
$$-\frac{\hbar^2}{2m^*} \frac{d^2 \psi_n(x)}{dx^2} + V(x) \psi_n(x) = E_n \psi_n(x)$$

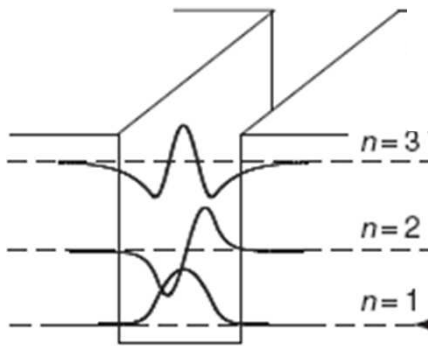
$$E_n = \frac{\hbar^2 n^2}{8m^* L^2} \quad \text{ENERGIA QUANTIZZATA}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

Soluzione nella  
 direzione di confinamento;  
 nelle altre direzioni "moto libero"

$$(n = 1, 2, 3, \dots, \infty)$$





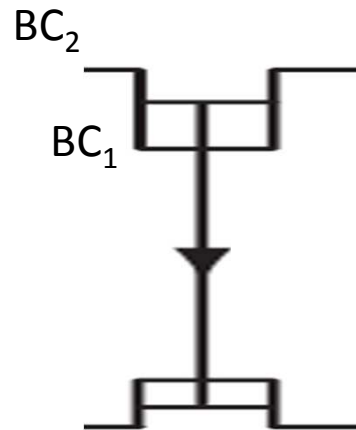
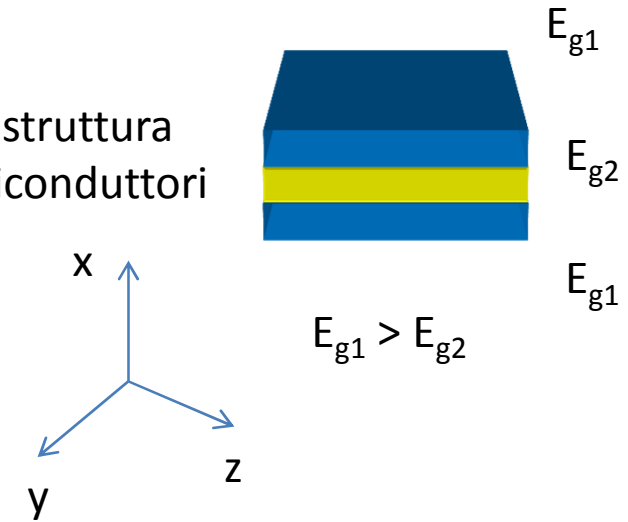
Il moto e' vincolato lungo x  
Buca quantica finita

Onde evanescenti

Numero finito di stati confinati

Soluzione numerica Eq. Schrodinger → approx. Buca infinita

Eterostruttura  
di semiconduttori



Per gli elettroni

Il fondo della buca pot. e' dato dal livello minimo di BC di un semiconduttore  
Il top e' dato dal livello minimo di BC dell'altro semiconduttore

Per le buche

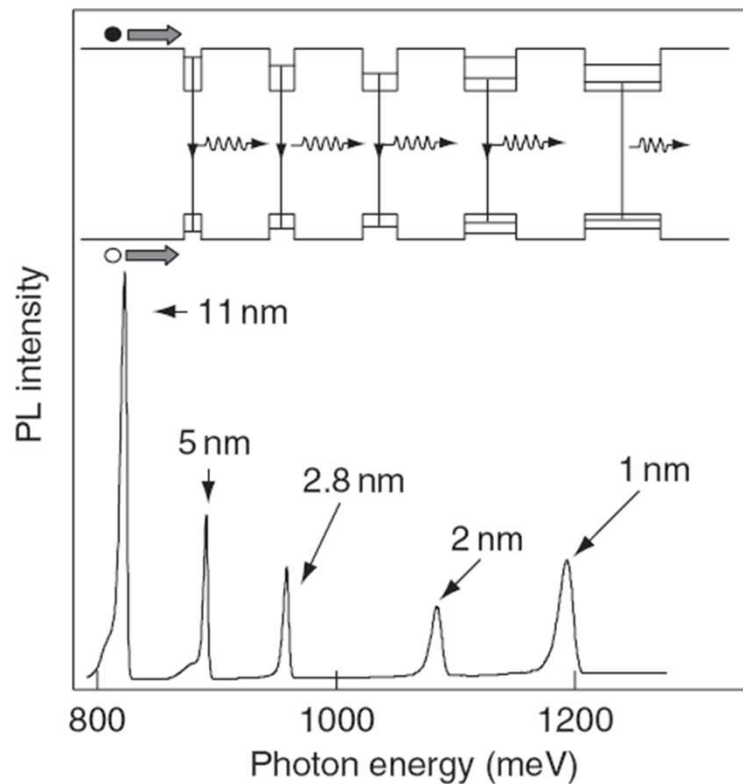
Analoghe considerazioni con BV



Il gap del semiconduttore "confinato" e' dato da

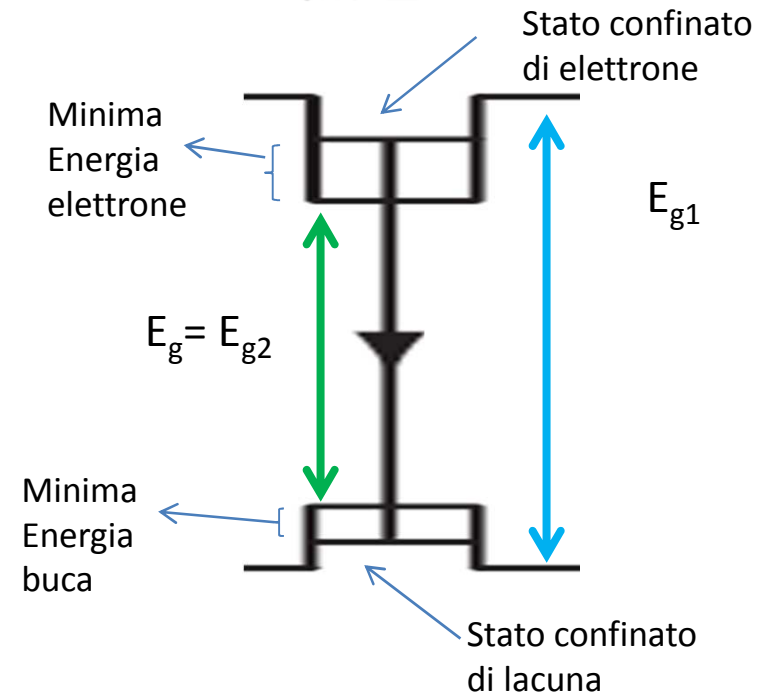
$$E_g^{ef} = E_g + \frac{h^2}{8m_e^*L^2} + \frac{h^2}{8m_h^*L^2}$$

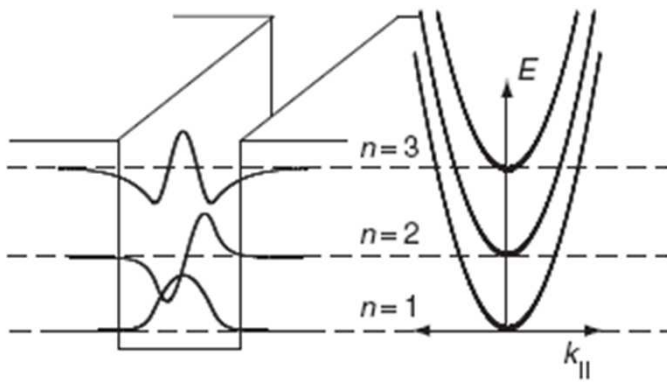
Al variare della larghezza L cambia il gap



$$E_n = \frac{h^2 n^2}{8m^* L^2}$$

Per n=1





Il confinamento e' perpendicolare al piano  
Lungo il piano: moto libero

$$E = \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

Energia totale (elettroni o lacune con massa efficace opportuna)

$$E_{n,k_{\parallel}} = \frac{\hbar^2 n^2}{8m^* L^2} + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

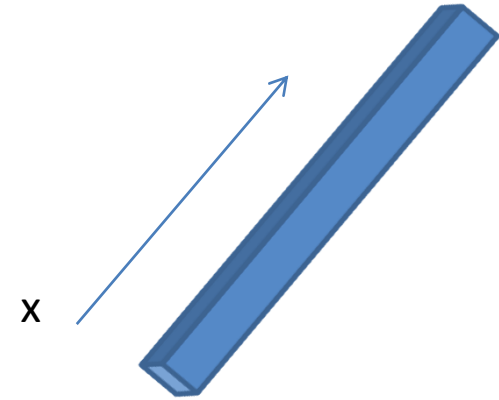
Continuo di stati sovrapposti agli stati quantizzati

ENERGIA QUANTIZZATA

SOTTOBANDE

# CONFINAMENTO QUANTICO : QUANTUM WIRE - 1D

Ragionamenti analoghi ai precedenti  
moto confinato su 2 direzioni



$$E_{n,m,k_x} = \frac{\hbar^2 n^2}{8m^* L_z^2} + \frac{\hbar^2 m^2}{8m^* L_y^2} + \frac{\hbar^2 k_x^2}{2m^*} \quad (n, m = 1, 2, 3, \dots)$$

ENERGIA QUANTIZZATA

SOTTOBANDA

$$\varepsilon = \varepsilon_{i,j} + \hbar^2 k^2 / 2m ; \quad \psi(x,y,z) = \psi_{i,j}(x,y) e^{ikz}$$

Densita' degli stati

$$D(\varepsilon) = \sum_{i,j} D_{i,j}(\varepsilon)$$

$$D_{i,j}(\varepsilon) = \frac{dN_{i,j}}{dk} \frac{dk}{d\varepsilon} = (2)(2) \frac{L}{2\pi} \left[ \frac{m}{2\hbar^2 (\varepsilon - \varepsilon_{i,j})} \right]^{1/2} = \frac{4L}{h v_{i,j}} \quad \text{for } \varepsilon > \varepsilon_{i,j}$$

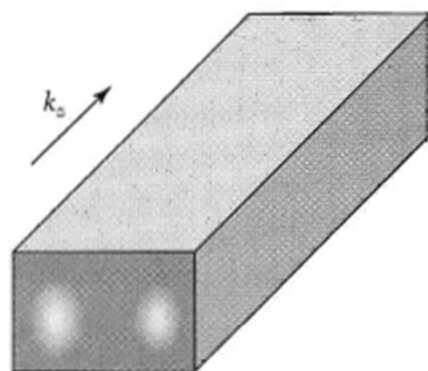
$$= 0$$

$$\text{for } \varepsilon < \varepsilon_{i,j}$$

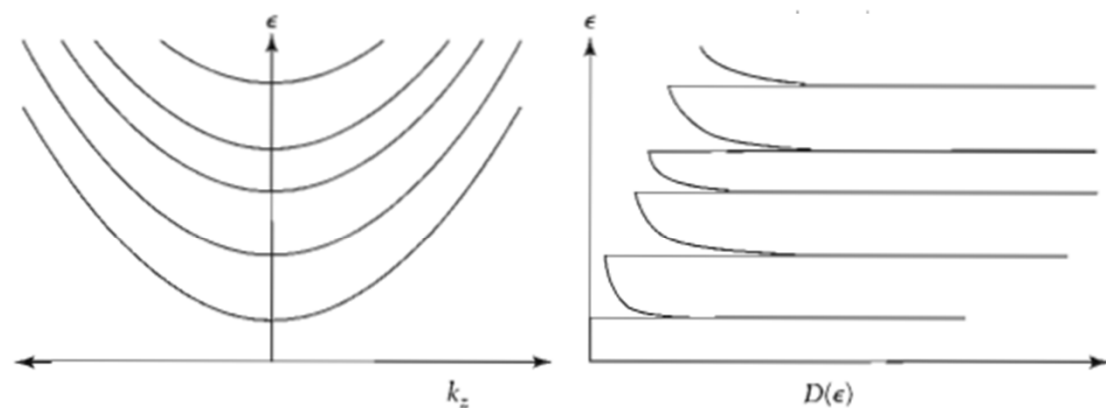
$v_{i,j}$  is the velocity of the electron in the  $i,j$  subband

$$D_{i,j}(\epsilon) = \frac{dN_{i,j}}{dk} \frac{dk}{d\epsilon} = (2)(2) \frac{L}{2\pi} \left[ \frac{m}{2\hbar^2 (\epsilon - \epsilon_{i,j})} \right]^{1/2} = \frac{4L}{h\nu_{i,j}} \quad \text{for } \epsilon > \epsilon_{i,j}$$

$$= 0 \quad \text{for } \epsilon < \epsilon_{i,j}$$



the density of states diverges as  $(\epsilon - \epsilon_{i,j})^{-1/2}$



**van Hove singularities.**

**Figure 9** Schematic of a rectangular quasi-one-dimensional wire, along with the dispersion relations and the density of states of the 1D subbands. The peaks in the density of states at the subband thresholds are called Van Hove singularities. The probability density for the  $i = 2, j = 1$  state is shown as a gray scale on the cross section of the wire.

Le proprietà elettriche ed ottiche sono affette dalla densità degli stati

Regola d'oro di Fermi per le transizioni tra stato  $i$  ed  $f$

$$w_{i \rightarrow j} = (2\pi/\hbar) \left| \langle j | e\mathbf{E} \cdot \mathbf{r} | i \rangle \right|^2 \delta(\epsilon_j - \epsilon_i - \hbar\omega)$$

assorbimento

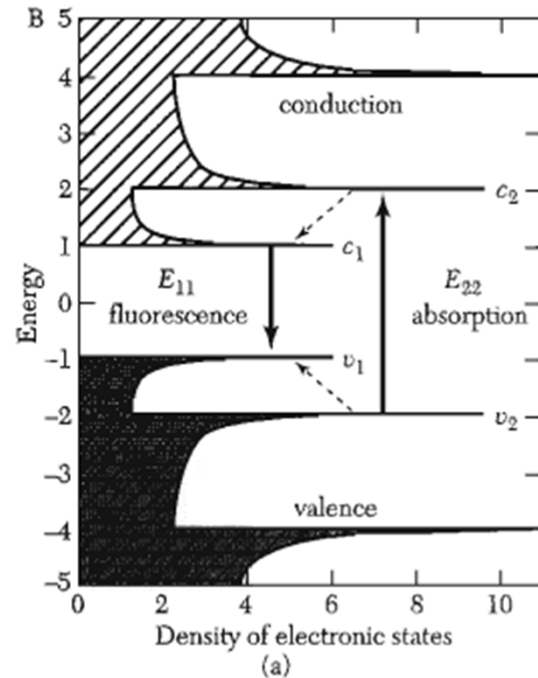
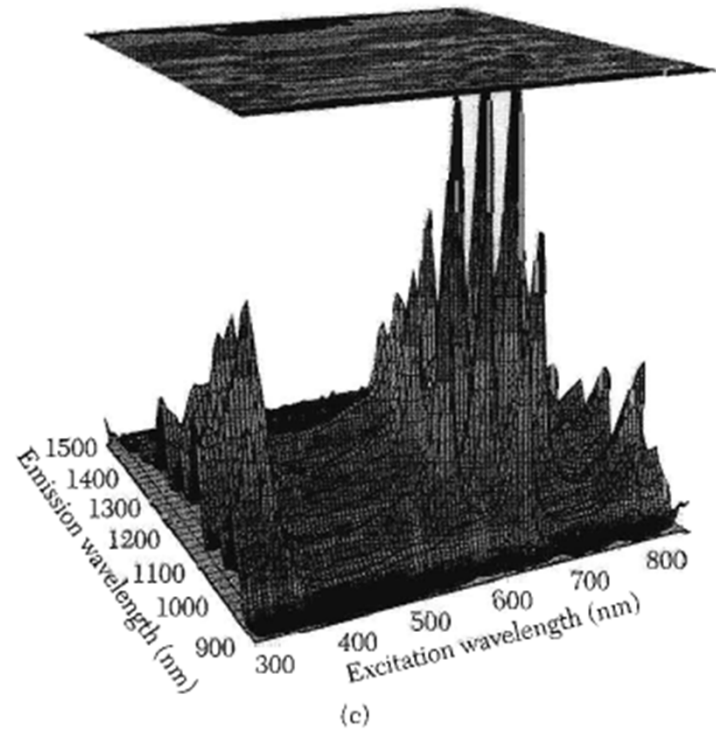
$$w_{j \rightarrow i} = (2\pi/\hbar) \left| \langle j | e\mathbf{E} \cdot \mathbf{r} | i \rangle \right|^2 \delta(\epsilon_i - \epsilon_j + \hbar\omega) + (4\alpha\omega_{ji}^3/c^2) \left| \langle j | \mathbf{r} | i \rangle \right|^2$$

emissione

where  $\omega_{ji} = (\epsilon_j - \epsilon_i)/\hbar$  and  $\alpha$  is the fine structure constant

absorption is proportional to the joint density of all initial and final states separated by an energy  $\hbar\omega$ , weighted by the dipole matrix element and the occupation factors of the states. The Fermi functions indicate that absorption only occurs when the initial state  $i$  is filled and the final state  $j$  is empty.

## Nanotubi di Carbonio



**Figure 10** (a) The density of states for a semiconducting carbon nanotube as a function of energy. The Van Hove singularities are seen in the STM tunneling spectra of a nanotube shown in (b). In (c), the emission intensity is plotted as a function of the emission wavelength and the excitation wavelength. Peaks in the intensity are observed when the absorption and emission energies correspond to those shown in the diagram (a). Different peaks correspond to nanotubes with different radii and chirality. [After Bachilo et al. (a and c) and C. Dekker (b).]

## CONFINAMENTO QUANTICO : QUANTUM DOTS - 0D

A system of electrons fully confined in all three dimensions will have discrete charge and electronic states, as do atoms and molecules. They are often called artificial atoms or **quantum dots** to reflect the importance of quantization phenomena on their properties.

Ragionamenti analoghi ai precedenti  
moto confinato su 3 direzioni



CUBOIDE

$$E_{n,m,l} = \frac{h^2 n^2}{8m^* L_z^2} + \frac{h^2 m^2}{8m^* L_y^2} + \frac{h^2 l^2}{8m^* L_x^2} \quad (n, m, l = 1, 2, 3, \dots)$$

ENERGIA QUANTIZZATA

NO SOTTOBANDE

In virtu' della simmetria sferica  
 si possono separare la parte radiale ed angolare



sfera

$$\varepsilon_{n,l,m} = \varepsilon_{n,l} ; \quad \psi(r,\theta,\phi) = Y_{l,m}(\theta,\phi)R_{n,l}(r)$$

Armoniche sferiche

Funzioni d'onda radiali

Per potenziale  $V=0$  per  $r < R$  e  $V=\infty$  per  $r > R$

$$\varepsilon_{n,l} = \hbar^2 \beta_{n,l}^2 / (2m^* R^2) ,$$

$$R_{n,l}(r) = j_l(\beta_{n,l} r/R) , \quad r < R$$

Funzione sferica di Bessel di ordine l

n-esimo zero della funzione di Bessel l

$$\beta_{0,0} = \pi \text{ (1S)}, \beta_{0,1} = 4.5 \text{ (1P)}, \beta_{0,2} = 5.8 \text{ (1D)}, \beta_{1,0} = 2\pi \text{ (2S)} \quad \beta_{1,1} = 7.7 \text{ (2P)}$$

I numeri tra parentesi indicano gli equivalenti degli stati atomici



## Densita' degli stati 3D

Relazione lunghezza d'onda - dimensione spaziale in una delle 3 dimensioni

Lunghezza d'onda di De Broglie  $\lambda = \frac{h}{p_x}$

$$\frac{L}{\lambda} = n_x$$

$$Lp_x = hn_x$$

$$Ldp_x = h$$

Per una variazione unitaria di  $n_x$   
si ha l'incremento di momento

In un cubo: 3D  $\longrightarrow L^3 dp_x dp_y dp_z = h^3$

per un cubo unitario la variazione incrementale e'  $dp_x dp_y dp_z = h^3$

corrisponde ad un dato stato energetico  $n_x, n_y, n_z$

Volume associato allo stato nello spazio k  $\longrightarrow h^3$

## Densita' degli stati 3D

Volume associato allo stato nello spazio k  $\longrightarrow h^3$

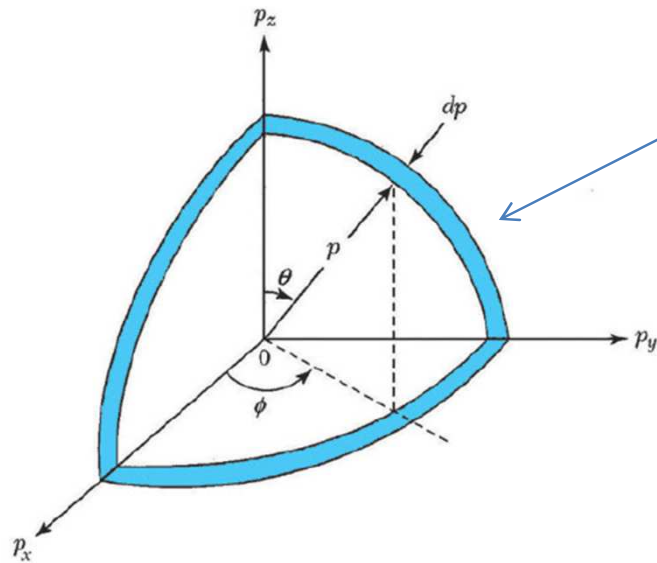


Fig. 1 The momentum space in spherical coordinates.

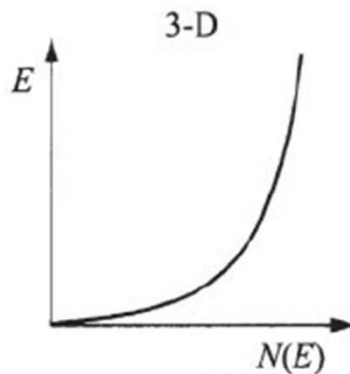
volume  $\longrightarrow 2(4\pi p^2 dp)/h^3$   
 $4\pi p^2 dp$

Numero di stati contenuti tra p e p+dp  
 con molteplicita' di spin

$$E = \frac{p^2}{2m_n}$$

Poiche'

$$p = \sqrt{2m_n E}$$

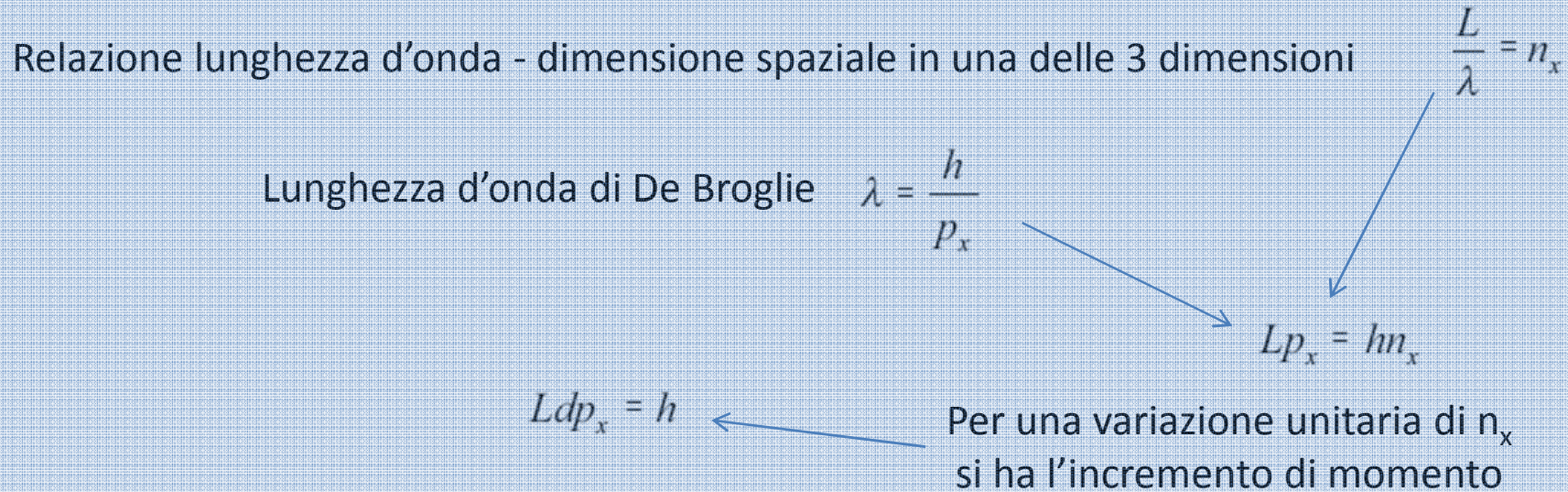


$$N(E)dE = \frac{8\pi p^2 dp}{h^3} = 4\pi \left( \frac{2m_n}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$



$$N(E) = 4\pi \left( \frac{2m_n}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

## Densita' degli stati 2D



In un quadrato: 2D  $\longrightarrow L^2 dp_x dp_y = h^2$

la direzione z da' "quantizzazione"

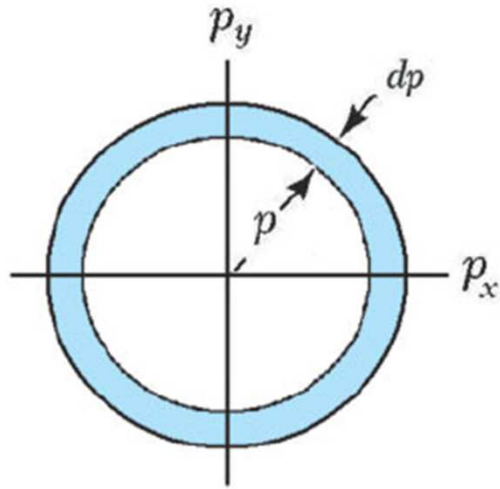
per un quadrato unitario la variazione incrementale e'  $dp_x dp_y = h^2$

corrisponde ad un dato stato energetico  $n_x, n_y$

Area associata allo stato nello spazio k  $\longrightarrow h^2$

## Densita' degli stati 2D

Area associata allo stato nello spazio k  $\longrightarrow h^2$



area  
 $2\pi p dp$ .

$$\longrightarrow \frac{2(2\pi p dp)}{h^2}$$

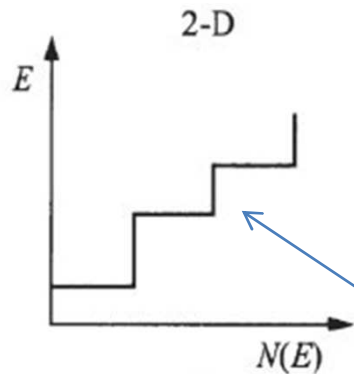
Numero di stati contenuti tra  $p$  e  $p+dp$   
con molteplicita' di spin

Fig. 3 The momentum space in circular coordinates.

Poiche'

$$E = \frac{p^2}{2m_n}$$

$$p = \sqrt{2m_n E}$$



$$N(E)dE = \frac{4\pi p dp}{h^2} = 4\pi \left( \frac{m_n}{h^2} \right) dE$$

$$N(E) = \frac{4\pi m_n}{h^2} = \frac{m_n}{\pi \hbar^2}$$

I salti corrispondono  
alla variazione di stato confinato lungo  $z$

## Densita' degli stati 1D

Relazione lunghezza d'onda - dimensione spaziale in una dimensione

Lunghezza d'onda di De Broglie  $\lambda = \frac{h}{p_x}$

$$\frac{L}{\lambda/2} = n_x$$

$$2Lp_x = hn_x$$

$$2Ldp_x = h$$

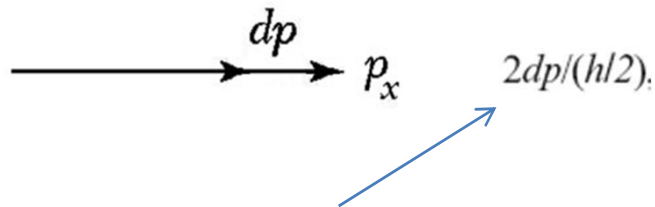
Per una variazione unitaria di  $n_x$  si ha l'incremento di momento

In un filo: 1D

le direzioni y,z danno "quantizzazione"

per un filo unitario la variazione incrementale e'  $dp_x = h/2$

corrisponde ad un dato stato energetico  $n_x$



Numero di stati contenuti tra  $p$  e  $p+dp$  con molteplicita' di spin

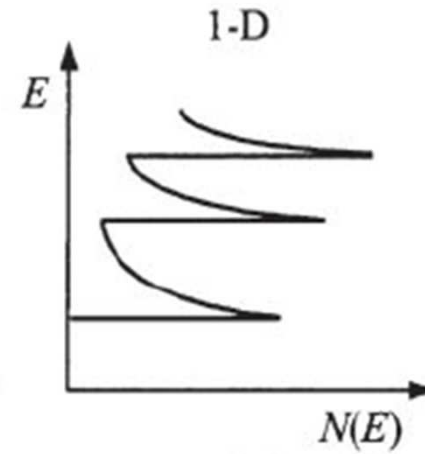
$$N(E)dE = \frac{2dp}{h/2} = 2 \left( \frac{2m_n}{E} \right)^{1/2} \frac{1}{h} dE = \frac{1}{\pi} \left( \frac{2m_n}{\hbar^2} \right)^{1/2} \frac{1}{E^{1/2}} dE$$

$$N(E) = \frac{1}{\pi} \left( \frac{2m_n}{\hbar^2} \right)^{1/2} \frac{1}{E^{1/2}}$$

## Densita' degli stati 1D

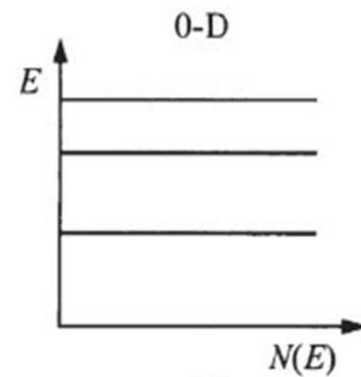
$$N(E)dE = \frac{2dp}{h/2} = 2 \left( \frac{2m_n}{E} \right)^{1/2} \frac{1}{h} dE = \frac{1}{\pi} \left( \frac{2m_n}{\hbar^2} \right)^{1/2} \frac{1}{E^{1/2}} dE$$

$$N(E) = \frac{1}{\pi} \left( \frac{2m_n}{\hbar^2} \right)^{1/2} \frac{1}{E^{1/2}}$$

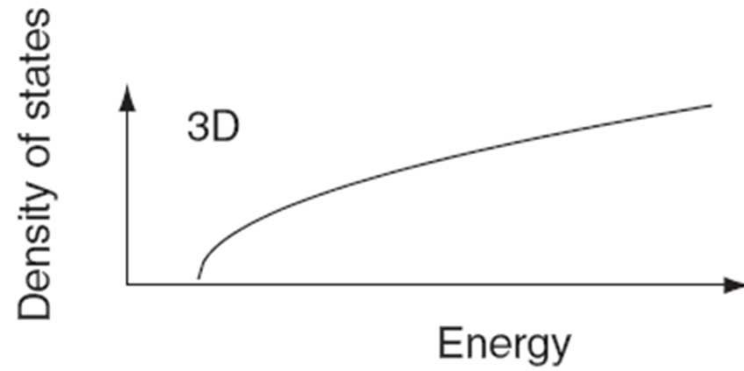


## Densita' degli stati 0D

le direzioni x, y, z danno "quantizzazione"

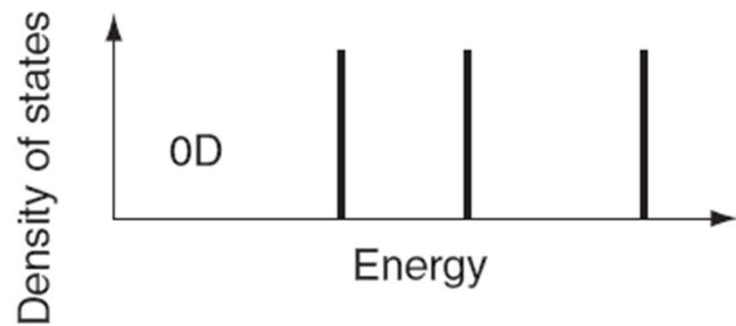


## DENSITA' DEGLI STATI



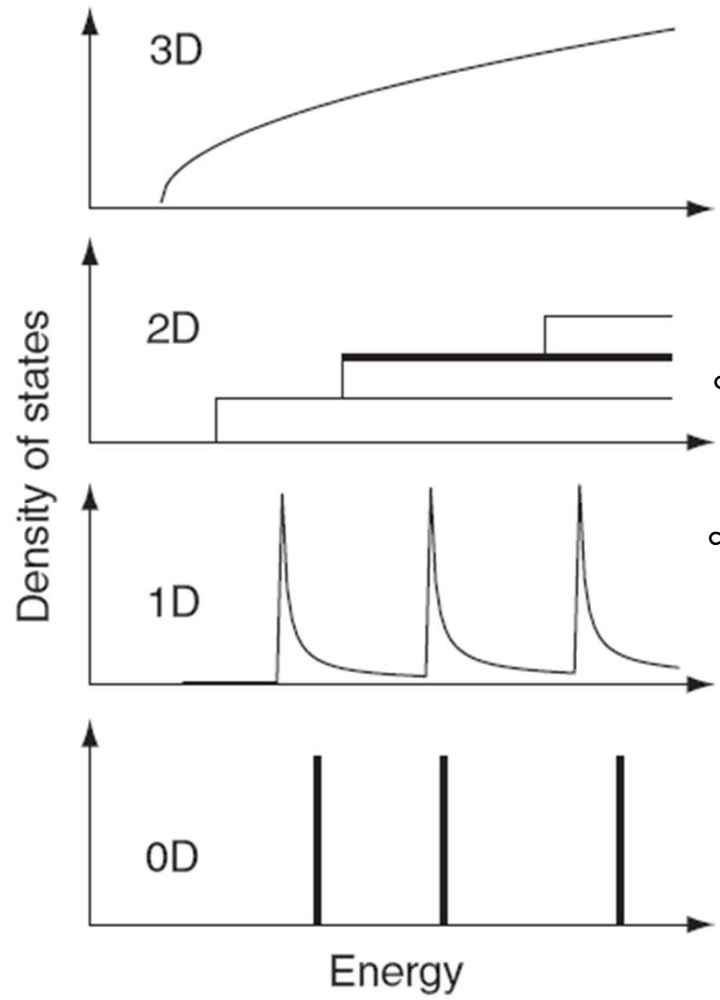
In un semiconduttore 3D

Densita' degli stati  $\propto E^{1/2}$



$$E_{n,m,l} = \frac{h^2 n^2}{8m^* L_z^2} + \frac{h^2 m^2}{8m^* L_y^2} + \frac{h^2 l^2}{8m^* L_x^2}$$

Manca la distribuzione continua  
Densita' degli stati discreta



In generale  
 Densita' degli stati con dipendenza  
 dall'energia  $E^{(n-2)/2}$

2D, 1D

$\propto E^{(2-2)/2} = E^0$

$\propto E^{(1-2)/2} = E^{-1/2}$

n = dimensioni presenti

Sottobande paraboliche

Influenza su proprieta'  
 elettroniche ed ottiche



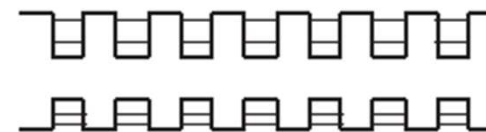
## SUPERRETICOLI DI SEMICONDUCTORI

Strutture periodiche di buche quantiche

La larghezza della barriera stabilisce  
la distribuzione dei livelli creando delle  
bande con trasporto di carica nella struttura



SUPERRETICOLO



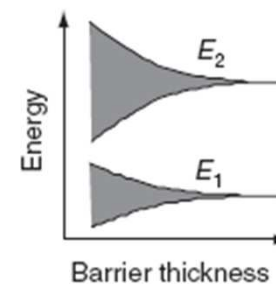
(a)

CONFINAMENTO



(b)

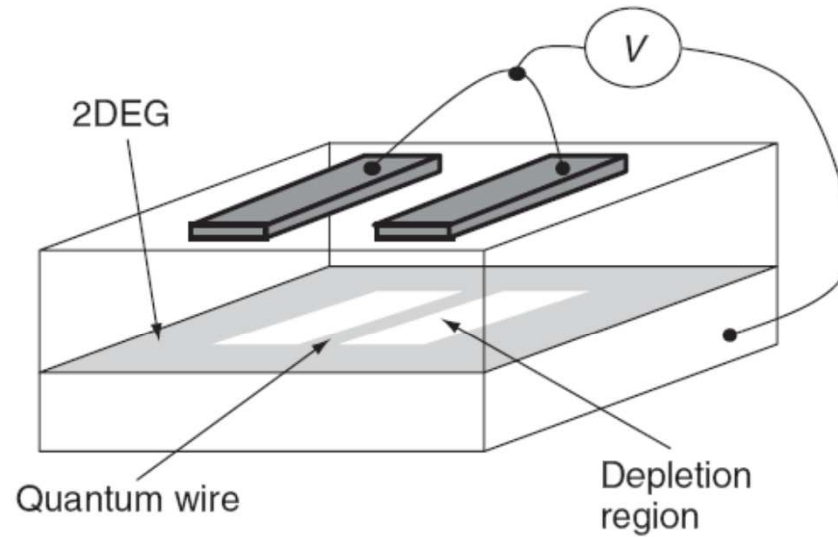
TUNNELLING



SUPERLATTICE

MULTIPLE  
QUANTUM  
WELL

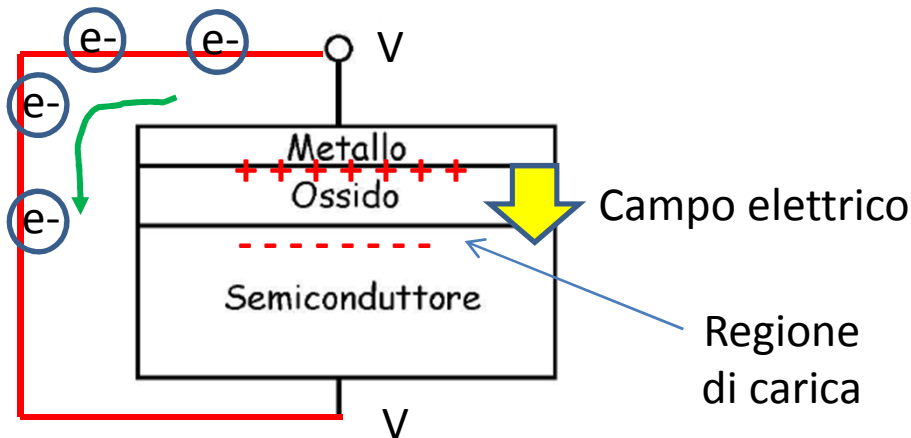
**Figure 3.12** The form of the confined energy states in (a) a multiple quantum well and (b) a superlattice; (c) shows how the discrete states in a multiple quantum well evolve into superlattice minibands as the barrier thickness is reduced



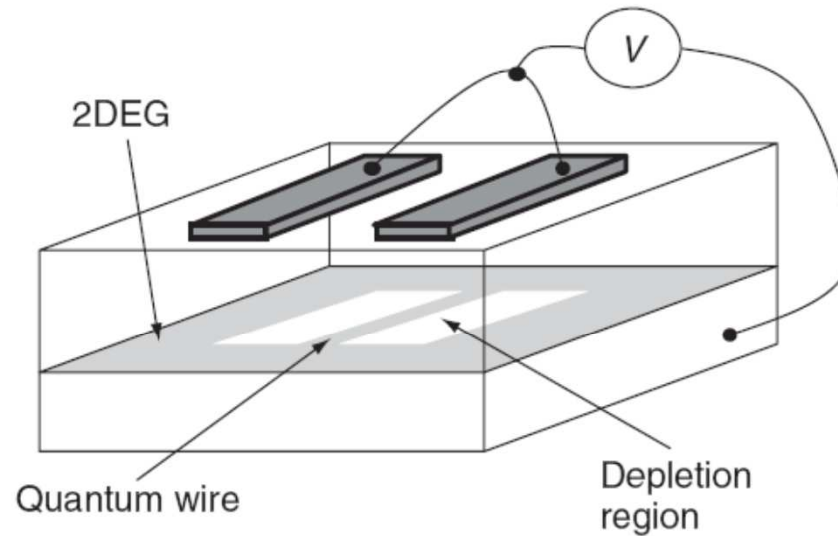
QUANTUM WIRE  
ELETTROSTATICO

**Figure 3.17** A schematic diagram of a split gate quantum wire. The electrons remaining below the gap between the gates form a quantum wire. A 2DEG remains in the regions away from the gates

Puo' essere ottenuto con un dispositivo MOS con due gate opportunamente disegnati



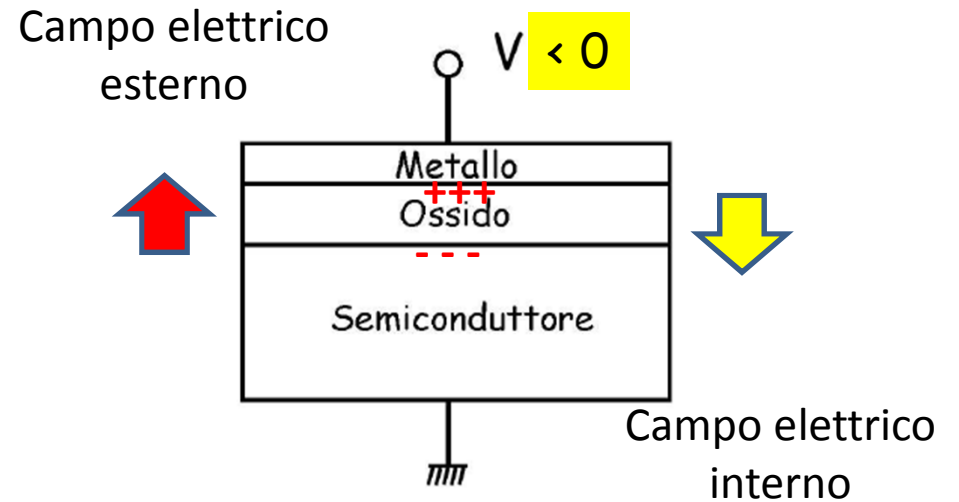
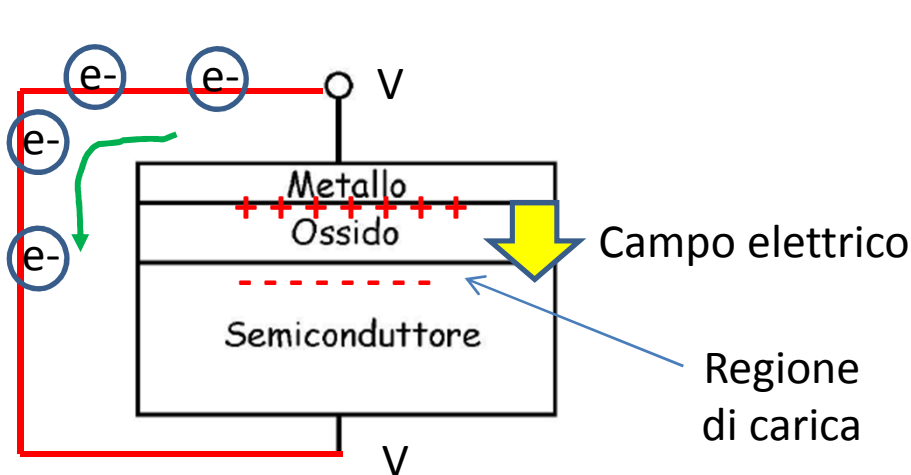
Distribuzione di carica senza applicazione di tensione esterna

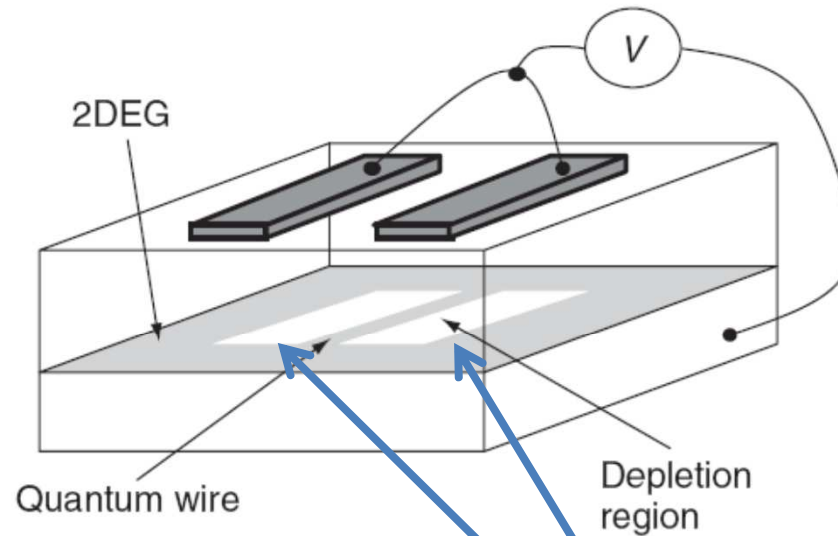


**Figure 3.17** A schematic diagram of a split gate quantum wire. The electrons remaining below the gap between the gates form a quantum wire. A 2DEG remains in the regions away from the gates

applicando tensione esterna negativa

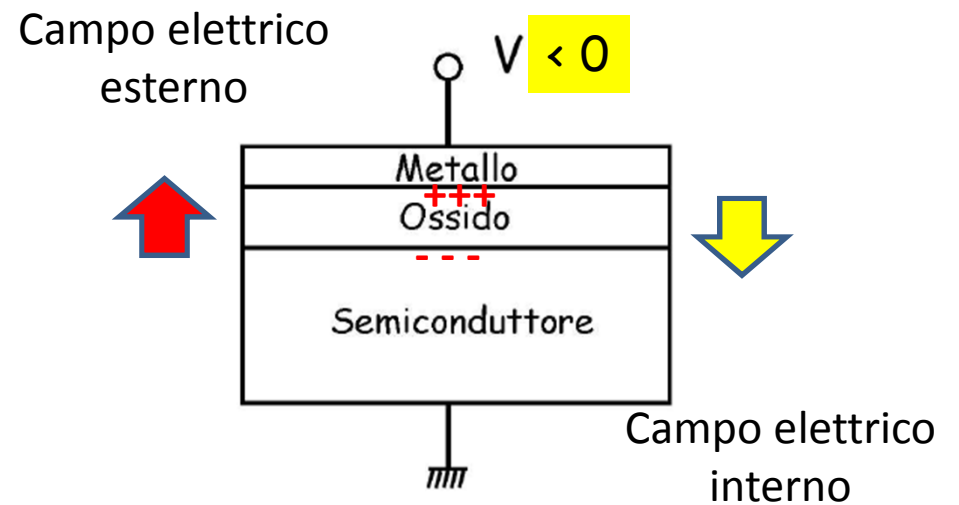
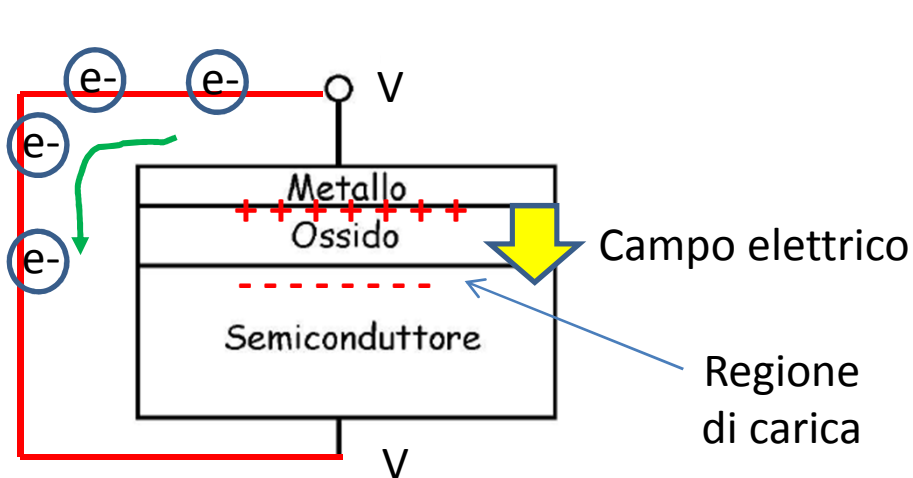
RIDUZIONE CARICA ACCUMULATA





**Figure 3.17** A schematic diagram of a split gate quantum wire. The electrons remaining below the gap between the gates form a quantum wire. A 2DEG remains in the regions away from the gates

RIDUZIONE CARICA ACCUMULATA



## Il trasporto di carica balistico in un quantum wire

Trasporto  
balistico



Le piccole dimensioni impediscono processi di scattering:  
portatore-fonone  
portatore-impurezze

Nel trasporto la fase del portatore di carica e' conservata

La corrente dipende da:

$$\bar{J}_n = -e n \bar{v}_d$$

Velocita'

Numero portatori → Densita' degli stati (massa eff.densita' degli stati)

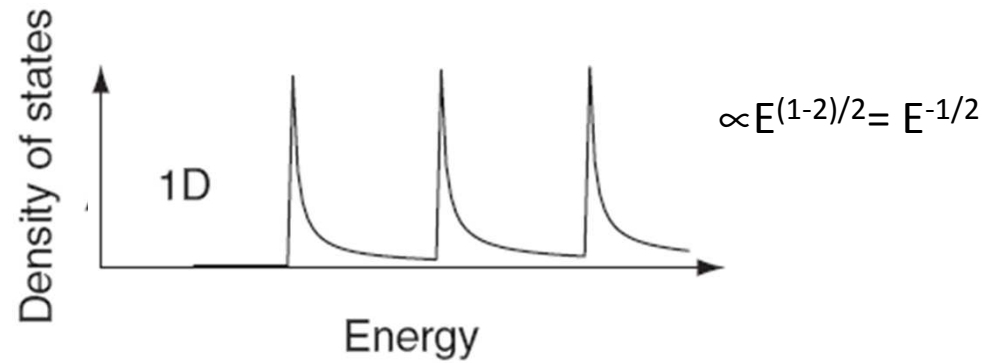
$$\begin{aligned} n(T) &= \int_{E_C}^{\infty} D_C(E) F(E) dE \\ &= \int_{E_C}^{\infty} \gamma \left( \frac{m_{dn}^*}{m_o} \right)^{3/2} (E - E_C)^{1/2} \exp \left[ -\frac{E - E_F}{kT} \right] dE \\ &= \frac{1}{2} \gamma \left( \frac{m_{dn}^* kT}{m_o} \right)^{3/2} \pi^{1/2} \exp \left[ -\frac{E_C - E_F}{kT} \right] \end{aligned}$$

## QUANTUM WIRE : 1D

Velocita'  $v = \sqrt{2E/m^*}$

Densita' stati  $\propto 1/\sqrt{E}$

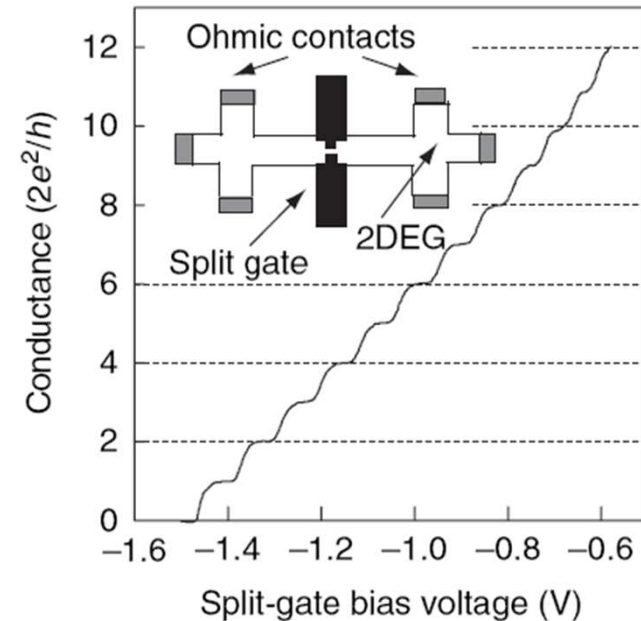
Per ogni sottobanda  
si ottiene conduttanza  $\propto 2e^2/h$



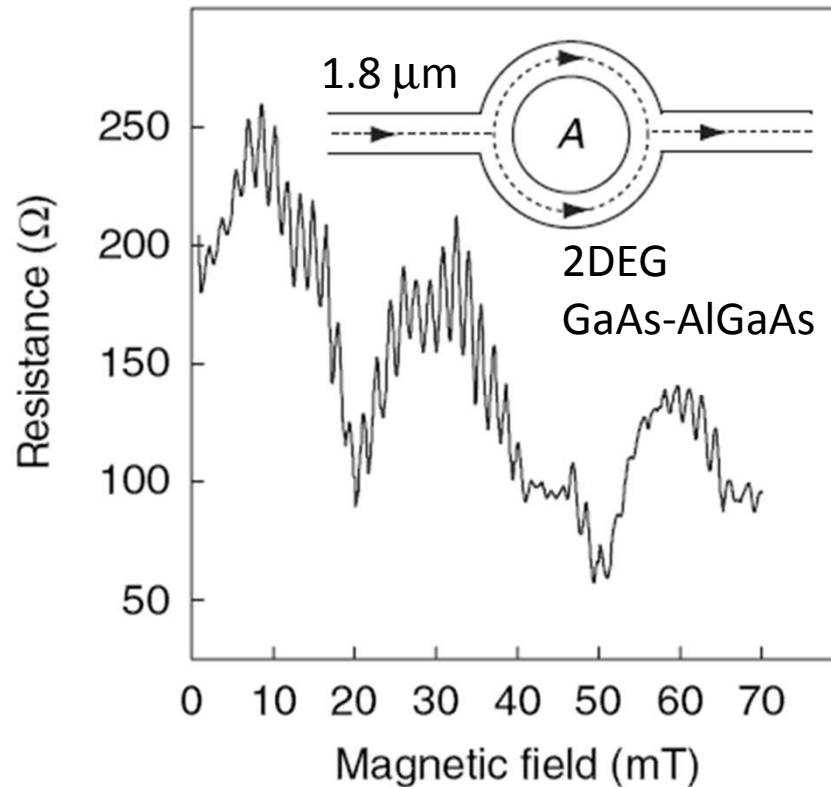
### Dipendenza a step

**Figure 3.31** Quantised conductance steps in a 400 nm long electrostatically defined quantum wire measured at a temperature of 17 mK. The wire is produced by a split gate formed on the surface of a modulation-doped GaAs–AlGaAs heterostructure. The inset shows the form of the electrical contacts. Data reproduced by permission of the American Institute of Physics from A. R. Hamilton, J. E. F. Frost, C. G. Smith, M. J. Kelly, E. H. Linfield, C. J. B. Ford, D. A. Ritchie, G. A. C. Jones, M. Pepper, D. G. Hasko and H. Ahmed, *Appl. Phys. Lett.* **60**, 2782 (1992)

$$E_{n,m,k_x} = \frac{\hbar^2 n^2}{8m^* L_z^2} + \frac{\hbar^2 m^2}{8m^* L_y^2} + \frac{\hbar^2 k_x^2}{2m^*} \quad (n, m = 1, 2, 3, \dots)$$



Le dimensioni del quantum wire possono essere controllate con la tensione di gate



**Figure 3.32** The Aharonov–Bohm effect in a 1.8 μm diameter ring, measured at a temperature of 280 mK. The inset shows the geometry of the structure. Data and figure reproduced from G. Timp, P. M. Mankiewich, P. deVegvar, R. Behringer, J. E. Cunningham, R. E. Howard, H. U. Baranger and J. K. Jain, *Phys. Rev. B* **39**, 6227 (1989). Copyright 1989 by the American Physical Society

Un quantum wire  
 Puo' essere splittato  
 in due per osservare  
 effetti di interferenza  
 tra portatori di carica

La funzione d'onda del  
 cammino top

Interferisce con quella  
 del cammino bottom

In presenza di campo B  
 Perpendicolare al piano

La differenza di fase e' data dall'ampiezza  
 del campo

Differenza di fase =  $2\pi$  quando il flusso di campo  $B \cdot A$  varia di  $h/e$

Al variare di B si cambia la conducibilita' : effetto Aharonov-Bohm

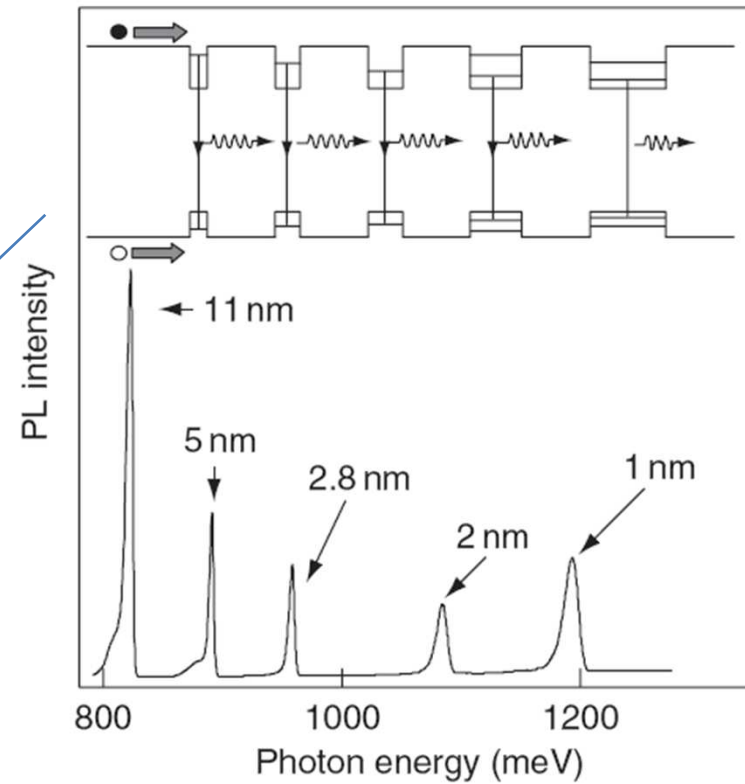
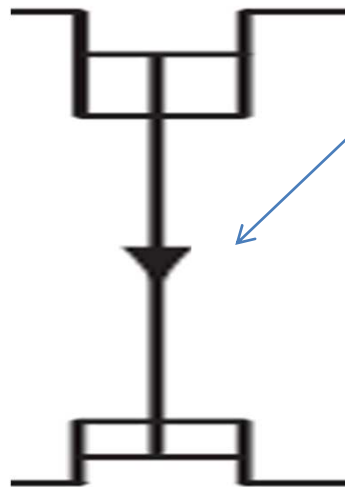
## Assorbimento ottico interbanda: quantum well (2D)

Dipende dalla densita' degli stati in banda di valenza e di conduzione

Dipende dalla separazione tra banda di valenza e di conduzione

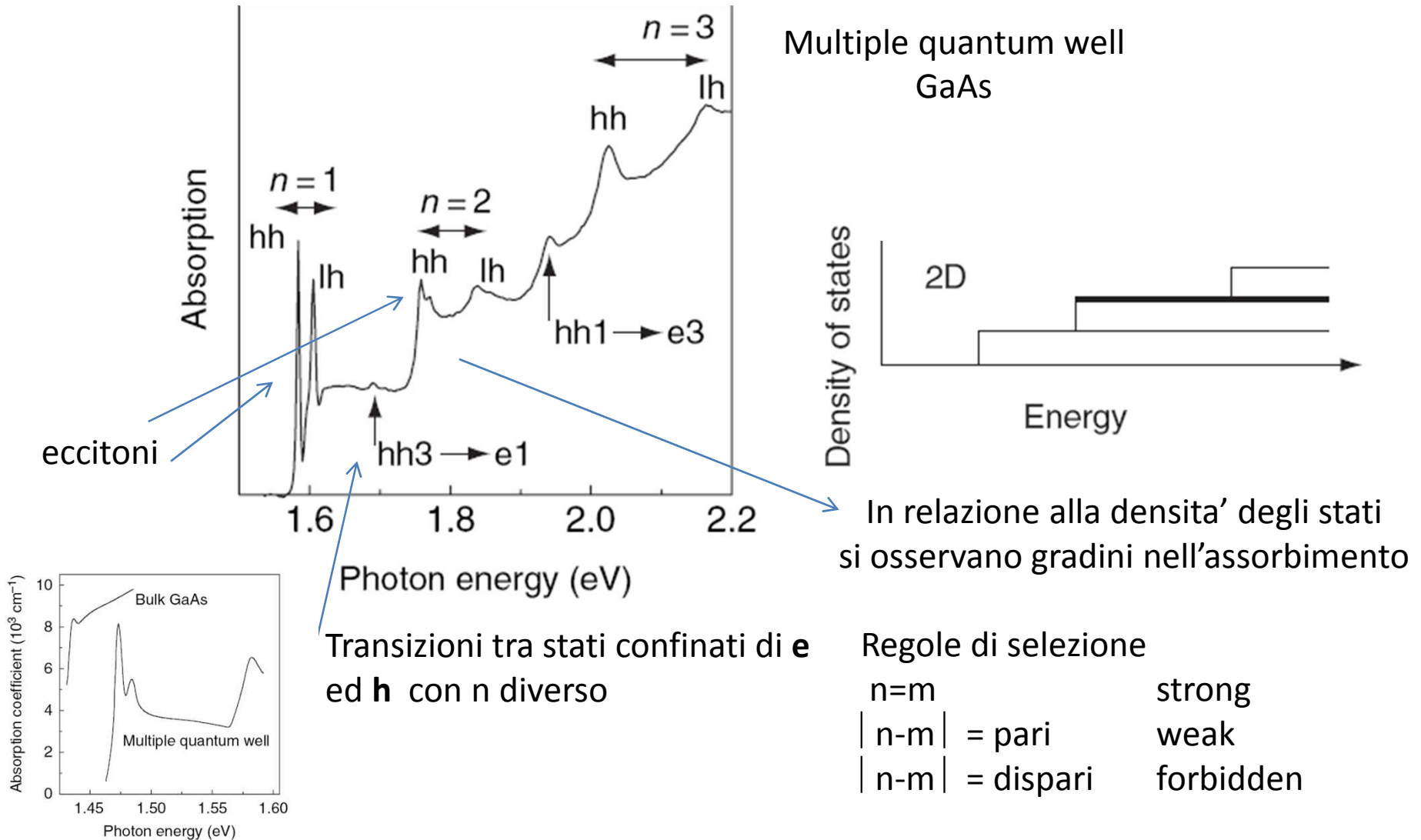
Gli stati eccitonici (e-h idrogenoide, livelli intragap) sono piu' legati a causa del confinamento (maggiore vicinanza tra le cariche) e quindi rivelabili anche ad alte T

$$E_g^{ef} = E_g + \frac{h^2}{8m_e^*L^2} + \frac{h^2}{8m_h^*L^2}$$





**Figure 3.33** Low-temperature absorption spectrum of a 40-period GaAs–AlAs multiple quantum well structure with 7.6 nm wide wells. The most intense features result from transitions between the  $n$ th ( $n = 1, 2, 3$ ) confined light hole (lh) and heavy (hh) hole states and identical index electron states. In addition, two weaker transitions are observed between the first and third heavy hole and electron states ( $hh3 \rightarrow e1$  and  $hh1 \rightarrow e3$ ). Data reproduced by permission of Taylor and Francis Ltd from A. M. Fox, *Contemp. Phys.* **37**, 111 (1996)



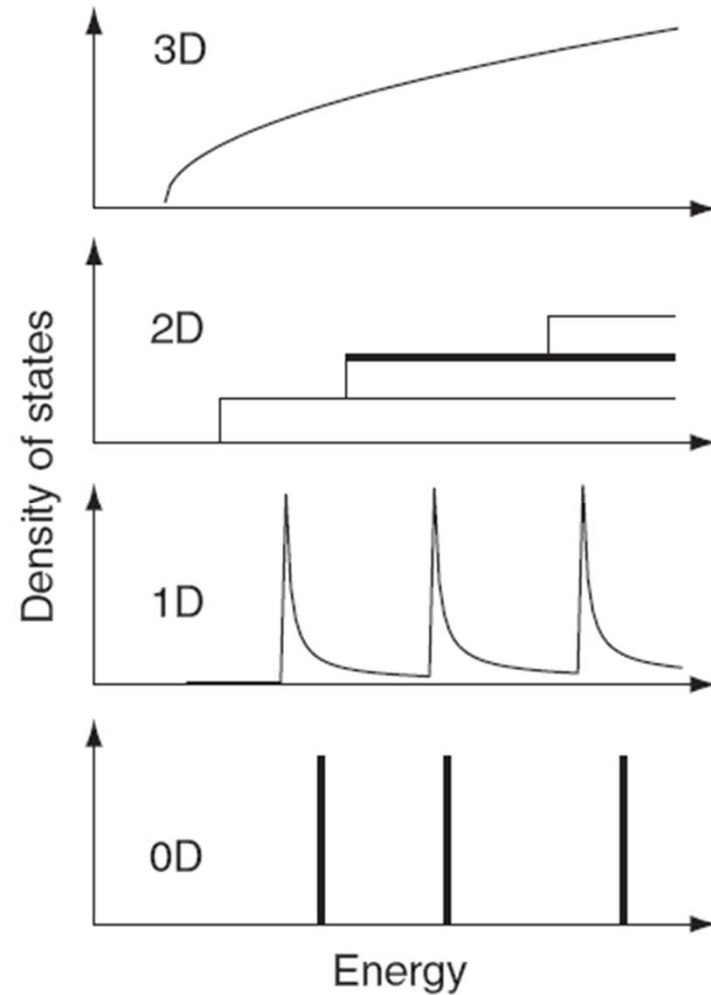
## Assorbimento ottico: quantum wire (1D)

Il confinamento in due direzioni da luogo a piu' stati eccitonici

Forti disomogeneita' indotte dalla crescita e variabilita' delle dimensioni dei nanofili

## Assorbimento ottico: quantum dots (0D)

Densita' degli stati discreta → analogia transizioni discrete tipo atomo



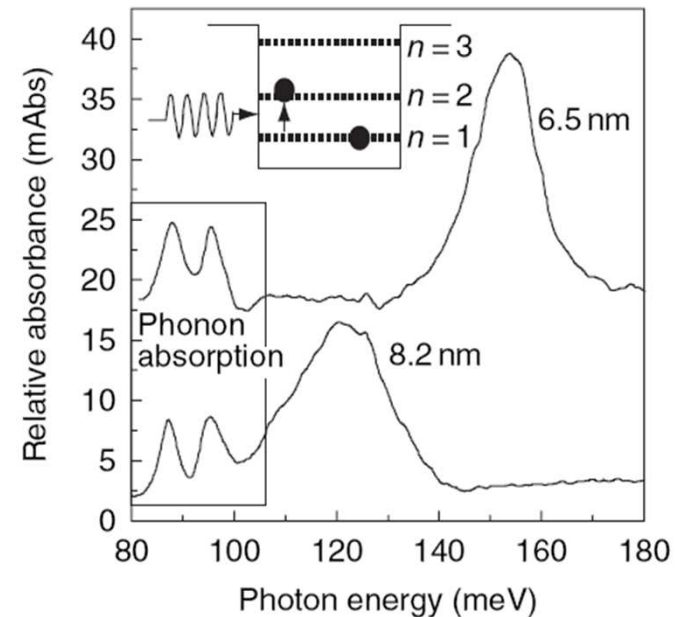
Assorbimento ottico intrabanda: quantum well (2D)

Sono coinvolti stati confinati di soli **e** o di soli **h**

NO EFFETTI ECCITONICI poiche' richiedono  $e/h$

Forte dipendenza dalla dimensione della nanostruttura

Separazione tra i livelli 10-200 meV



**Figure 3.35** Intraband absorption spectra of two 50-period GaAs multiple quantum well structures with wells of width 6.5 and 8.2 nm. Transitions occur between the  $n = 1$  and  $n = 2$  confined electron levels. The inset shows a schematic diagram of the absorption process. Data reproduced by permission of the American Institute of Physics from L. C. West and S. J. Eglash, *Appl. Phys. Lett.* **46**, 1156 (1985)

Nanocristalli di semiconduttore CdSe

Massa efficace in banda di conduzione  $m_e^* = 0.13 m$

Livelli energetici degli elettroni

$$\epsilon_{n,l} = (2.9 \text{ eV}/R^2)(\beta_{n,l}/\beta_{0,0})^2$$

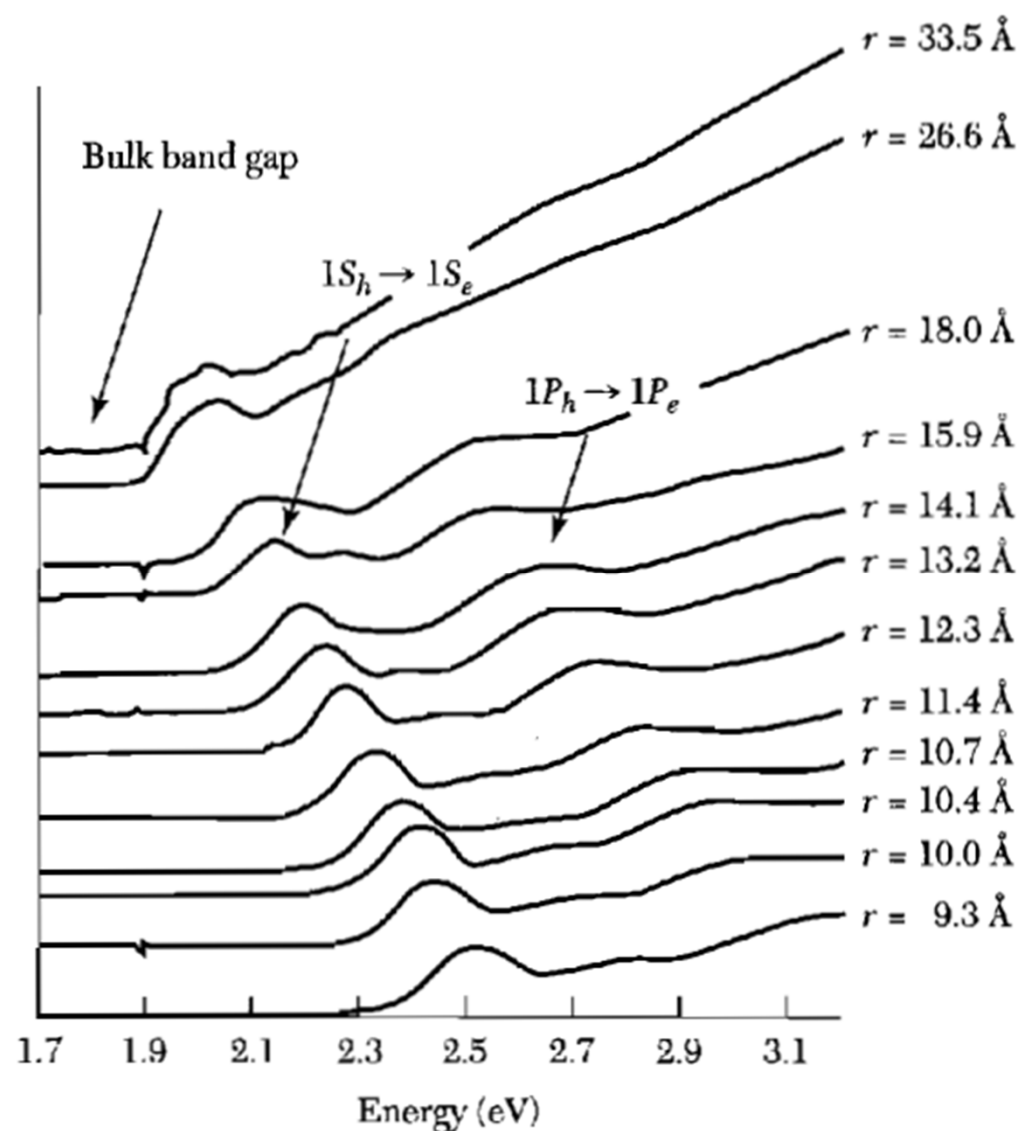
R = raggio nanoparticella  
in nanometri

$$R = 2 \text{ nm} \longrightarrow \epsilon_{0,1} - \epsilon_{0,0} = 0.76 \text{ eV.}$$

La separazione tra stati di banda di conduzione dipende da R

Analogamente la separazione tra stati di banda di valenza dipende da R

The 1S electron state increases in energy with decreasing  $R$ , while the 1S hole state decreases. The bandgap therefore grows and can be tuned over a wide range by changing  $R$ . This is shown in Fig. 19, where the absorption spectra of CdSe nanocrystals of different sizes are presented. For the smallest radii, the threshold for absorption shifts by nearly 1 eV from its bulk value. A similar shift is seen in the emission spectrum. The optical spectra of nanocrystals can be tuned continuously across the visible spectrum, making them useful in applications from fluorescent labeling to light-emitting diodes.



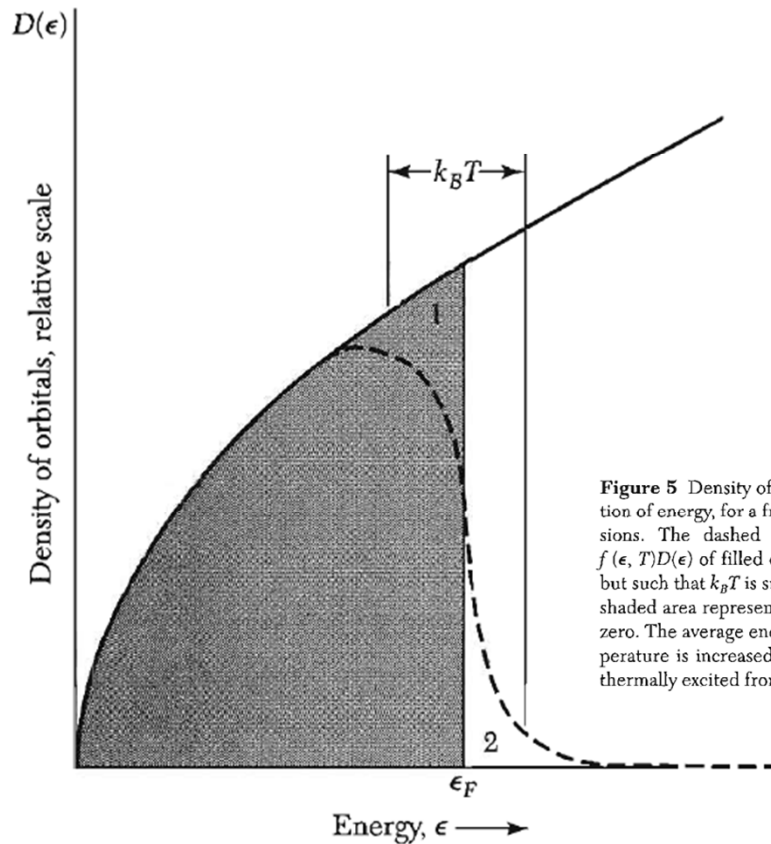
**Figure 19** Optical absorption spectra for a series of CdSe nanocrystal samples of different average radii. The lowest transition energy in the smallest nanocrystal sample is shifted by nearly 1 eV from the bulk bandgap. Two dominant transitions are labeled. (Courtesy of A. P. Alivisatos.)

$$D(\epsilon_F) = 3N/2\epsilon_F = 3N/2k_B T_F \quad \text{Si ottiene considerando che}$$

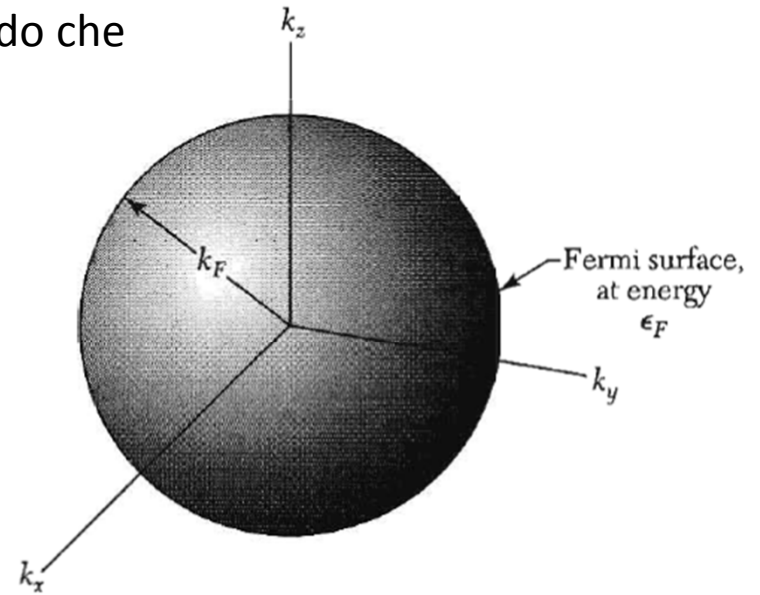
Per un gas di elettroni di concentrazione  $N/V$

$$\epsilon_F = \frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{2/3}$$

$$v_F = \left( \frac{\hbar k_F}{m} \right) = \left( \frac{\hbar}{m} \right) \left( \frac{3\pi^2 N}{V} \right)^{1/3}$$



**Figure 5** Density of single-particle states as a function of energy, for a free electron gas in three dimensions. The dashed curve represents the density  $f(\epsilon, T)D(\epsilon)$  of filled orbitals at a finite temperature, but such that  $k_B T$  is small in comparison with  $\epsilon_F$ . The shaded area represents the filled orbitals at absolute zero. The average energy is increased when the temperature is increased from 0 to  $T$ , for electrons are thermally excited from region 1 to region 2.



**Figure 4** In the ground state of a system of  $N$  free electrons the occupied orbitals of the system fill a sphere of radius  $k_F$ , where  $\epsilon_F = \hbar^2 k_F^2 / 2m$  is the energy of an electron having a wavevector  $k_F$ .

Orbitali di energia  $\leq \epsilon$ :

$$N = \frac{V}{3\pi^2} \left( \frac{2m\epsilon}{\hbar^2} \right)^{3/2}$$

$$D(\epsilon) \equiv \frac{dN}{d\epsilon} = \frac{V}{2\pi^2} \cdot \left( \frac{2m}{\hbar^2} \right)^{3/2} \cdot \epsilon^{1/2}$$

$$D(\epsilon) \equiv \frac{dN}{d\epsilon} = \frac{3N}{2\epsilon}$$