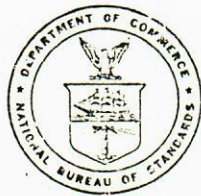NOTES TO SUPPLEMENT RESISTANCE THERMOMETER REPORTS
ON THE
INTERNATIONAL PRACTICAL TEMPERATURE SCALE OF 1968

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NOTES TO SUPPLEMENT RESISTANCE THERMOMETER REPORTS
ON THE INTERNATIONAL PRACTICAL TEMPERATURE SCALE OF 1968

The practical value of a standard resistance thermometer
depends upon the precision and convenience with which it can be
used to determine temperatures on a definite temperature scale.
These notes were prepared to supplement calibration reports because
of the increasing use of platinum resistance thermometers and the
increased precision sought with them. This revision of these notes
was motivated by the recent adoption of the new International Practical
Temperature Scale of 1968. This new scale replaces IPTS-48 which has
been the nearly universally used temperature scale and which has been
the scale promulgated by the National Bureau of Standards as the
official scale in this country. This is the first major change in the
temperature scale since 1927 and is the result of an international
effort to create a practical scale which both extends to a lower
temperature (down to 13.81 K) and is in closer agreement with the
thermodynamic scale.

I.   Temperature Scale

The International Practical Temperature Scale of 1968 (IPTS-68)
is based upon eleven defining fixed points[1] to which numerical
values have been assigned. Values between these fixed points are
defined by the indications of specified measuring instruments.
Specified formulas and tables are used to relate these indications to
temperature. Between -259.34 °C and 630.74 °C the instruments are
platinum resistance thermometers which have been calibrated at specified
defining fixed points to determine the constants in the formulas.

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[1]   See Appendix A.

over the range 0° to 630.74 °C temperatures may be calculated using the following expressions,[2]

$$t = t' + M(t') \tag{1}$$

$$t' = \frac{1}{\alpha} \left(\frac{R_t}{R_o} - 1\right) + \delta \left(\frac{t'}{100} - 1\right) \frac{t'}{100} \tag{2}$$

$$M(t') = .045 \left(\frac{t'}{100}\right) \left(\frac{t'}{100} - 1\right) \left(\frac{t'}{419.58} - 1\right) \left(\frac{t'}{630.74} - 1\right), \tag{3}$$

where t is the temperature in °C, R_t is the resistance at the temperature t, R_o is the resistance at 0 °C. The constants, α and δ, have values which are characteristic of the individual thermometer. The constant α, which is equivalent to $\frac{R_{100} - R_o}{100 \, R_o}$, may be determined from measurements of the thermometer resistor at the triple point of water (0.01 °C) and the steam point. The value of α must be greater than 0.003925 to satisfy one of the international requirements for a standard thermometer. The constant δ is determined from knowledge of α and measurements at a third fixed point, usually the freezing point of zinc. The addition of the small value represented by (3) serves to make the IPTS-68 conform more closely to the thermodynamic scale than can be done with only the simple quadratic of expression (2).

CAUTION: The values of A, B, and δ on the new 1968 scale are distinctly different from the corresponding values on the old 1948 or 1927 scale. The values of α and R_o are also different but only trivially so.

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[2] For the computation of tables it is suggested that expression (2) be replaced by the following equivalent formula which is given in the text of International Practical Temperature Scale of 1968,

$$R_t = R_o (1 + At' + Bt'^2),$$

where R_t is the resistance of the resistor of the thermometer at temperature t, and R_o is the resistance at 0 °C. A and B are constants which are related to α and δ of the Callendar formula by the identities,

$$A = \alpha (1 + \delta/100) \text{ and } B = -\alpha\delta/10^4.$$

Temperatures below 0 °C on the new 1968 scale are calculated using
a standard reference table which gives values of R_t/R_0 for a fictitious
"mean" standard thermometer. This reference table and a specified
deviation equation are combined to give values of R_t/R_0 for a particular
thermometer. The standard reference table used for IPTS-68 is referred
to as the "CCT-68" table.[3] It is convenient to use the symbol W_t
in place of R_t/R_0. For the special reference values of R_t/R_0 tabulated
in CCT-68 the special symbol W_t* is used. The table giving values of
W_t from 0 °C down to -182.962 °C for a particular thermometer may be
calculated from the following expressions,

$$W_t = W_t* + \Delta W_t \qquad (4)$$

$$\Delta W_t = A_4 t + C_4 t^3 (t - 100). \qquad (5)$$

Expression (5) is the specified deviation equation in the range 0 °C
to -182.962 °C. The constants A_4 and C_4 are also characteristic of the
individual thermometer. The value of A_4 is simply $\alpha - .0039259668$.
The value of C_4 is determined from measurements at the boiling point
of oxygen.

II. Measurement for the Determination of Temperature

A standard platinum resistance thermometer is constructed so
that precise measurements may be made of the resistance of its platinum
resistor. The resistor is made of very pure platinum wire, is compact
in form, and is contained in a protecting tube. At each end of the
resistor is a branch point to which are joined two leads, commonly
called "current" and "potential" leads. This construction makes it
possible to measure the resistance of the resistor alone (i.e. between
the branch points), independent of the resistance of the leads.

[3] See Appendix A

The resistance of the thermometer may be measured in various ways. The Wheatstone bridge and commutator of Mueller's design are commonly used in this country. The function of the commutator is to interchange the thermometer leads in such a way that the average of the resistances measured with the commutator in the N (normal) position and in the R (reverse) position is the resistance of the resistor of the thermometer. The resistance of the leads is completely eliminated by this technique only if the <u>difference</u> in the resistance of the two leads appearing in the arms of the bridge is the same when measurements are made in the N as when measurements are made in the R position. By making a series of four measurements, at equal time intervals, in the order N R R N, any change of the difference of the lead resistance is computable and the average value of the four measurements is independent on any linear drift in this difference. The resistance may also be measured with a potentiometer, a Kelvin-type double bridge, or one of the more recently developed bridges using inductive dividers.

The calibration report for a resistance thermometer gives the values of α, δ, and A_4 which were found to apply to the resistor of that particular thermometer. If the thermometer was calibrated at the oxygen point, the value of C_4 which was found is given for calculating temperatures below 0 °C. These constants are characteristic of the material in the particular resistor but do not depend on the magnitude of its resistance nor the units in which the resistance is measured.

For accurate determinations of temperature with a platinum thermometer it is necessary to refer to some known and reproducible

temperature (fixed point) which should be readily available whenever
it is needed. This fixed temperature is most commonly either the ice
point, or the triple point of water (0.01 °C). Measurement of the
resistance of the thermometer in a suitable ice bath will give R_O
directly, while a small correction, very nearly equivalent to 0.01
degree C, is necessary if a triple point cell is used.[4] The choice
between these two fixed points is largely on the basis of the accuracy
required. The triple point may be realized within ±0.0002 degrees C
while the accuracy of realizing the ice point is typically five to
ten times worse. The value of R_O is given in the report primarily to
enable the determination of any change occurring in R_O between the time
of calibration and the time the R_O is measured in the user's laboratory.
In addition to the errors of realizing the ice point or triple point,
the accuracy of such a check is limited by the uncertainty of the
difference in the resistance units used. The accuracy of temperatures
determined from values of R_t/R_O obviously is not limited by the uncertainty
of the resistance units if the values of R_t and R_O are determined in
the same units. Hence, precision temperature determinations with any
thermometer should be based upon a value of R_O determined from measure-
ments with the bridge which is to be used. Since temperature determinations
depend directly upon R_O, it should be redetermined frequently enough
to assure the user that he has a sufficiently reliable value when
measurements are made.

Because only ratios of resistances are required to determine
temperature, the units may be absolute ohms, international ohms, or
any arbitrary unit; but the calibrated bridge (or potentiometer) must

[4] H. F. Stimson, Precision Resistance Thermometry and Fixed Points,
     Temperature, Its Measurement and Control in Science and Industry,
     Vol. II, pp 141-168 (Reinhold Publishing Corp., New York, N. Y.,
     1955).

be self-consistent. A bridge that is self-consistent may be used for
precision temperature determinations using the constants reported for
the thermometer or using the table of R_t/R_0 values which has been
derived from the reported constants. It is advisable to calibrate the
bridge often enough to assure the user that the calibrations are re-
liable within the accuracy necessary for his work. Calibrations are
not difficult and may be made in the same place and under the same
laboratory conditions as the measurements for temperature determi-
nations. A method of calibrating the bridge is described in section
IV.

Certain precautions must be observed if reliable temperature
determinations are to be made with a resistance thermometer. The
thermometer coil must be immersed to a depth sufficient to prevent a
significant error from transfer of heat along the thermometer leads
and protecting tube. A check of the adequacy of the immersion in each
uniform constant-temperature bath may be made by varying the depth of
immersion of the thermometer and noting whether there is a change in
resistance. When a measuring current flows in the thermometer some
heating of the resistor results, consequently the same current should
be used in making measurements as was used in the calibration.
Sufficient time must be allowed, after the current is turned on, for
equilibrium to be established. The characteristic constants of a
thermometer may change as a result of changes in the dimensions of
the wire, strains in the wire, or the subjection of the thermometer
to excessive temperatures. It is particularly important that care
be taken to protect the thermometer from small mechanical shocks,
each of which strain the wire in the platinum resistor slightly to

produce small changes in its characteristics. If the measured resistance at a reliable fixed point is found to have changed by a significant amount, and the change cannot be attributed to the bridge, recalibration of the thermometer is advisable. The maximum change of the calibration indicated by a change in R_O may be estimated using the following expression:

$$(\text{Maximum Error at temperature } t) = t\left(\frac{Bt}{A} - 1\right)\frac{\Delta}{R_O},$$

where Δ is the change in R_O since calibration. This assumes that temperatures are being computed from values of R_t/R_O using the current value of R_O. The value for the maximum error is based on experience with standard platinum thermometers. It is not likely to be valid for others.

III. Tables

The labor involved in computing temperatures can be greatly reduced by making use of the table which is computed for each thermometer. The table relates resistance ratios to temperature over that part of the temperature range appropriate to both the thermometer and the calibration, or over the range requested. The table gives the resistance ratios at intervals of one degree; a linear interpolation will not introduce an additional error greater than the equivalent of 0.0001 degree at temperatures above -200 °C.

IV. Calibration of a Mueller-Type Bridge

The bridge may be calibrated in terms of any convenient unit of resistance such, for example, as a bridge unit which may be established by arbitrarily defining the sum of the resistance of the ten 1 ohm

resistors in the 1 ohm decade to be equal to 10 bridge units. However, it is decidedly preferable for the bridge to be calibrated in terms of absolute ohms by means of a suitable standardized resistor because one may then make checks on the stability of the thermometer. In any case it is essential that the calibrated bridge be self-consistent, that is, that the resistance change corresponding to each decade step be expressed in terms of a single unit of resistance. The resistance of a decade step is the change in the effective resistance of the variable arm of the bridge when that step is added to or taken from the bridge setting.

The method of calibration is to compare the resistance of each decade step with the resistance of the X (sum of all ten steps) of the next lower decade. This comparison is made by alternately including in the variable arm of the bridge either the step being calibrated or the X of the next lower decade. The lower decades of the bridge are used to make the variable arm balance an external resistance. From these alternate balances, a relation between the step being calibrated and the X of the next lower decade is obtained. The complete calibration consists of repeating this process for each decade step.

The comparisons require external resistors connected in such a manner that the resistance of the external circuit including all contacts will remain adequately constant during the short time required for each comparison. The external resistance may be that in a series circuit consisting of a resistance box, variable in 0.1 ohm steps up to 100 ohms, and a slide wire shunted to have a resistance slightly greater than 0.1 ohm. The shunted slide wire is convenient for getting the necessary external resistances for calibration of the 0.01 ohm and lower decades, and it is useful for the calibration

of the 0.1 ohm and higher decades. When both the slide wire and the
resistance box are connected to the bridge, the movable contact of
the slide wire is connected to the c post of the bridge and one end
of the slide wire to the C post. The resistance box is connected
between the other end of the slide wire and the T post of the bridge.
If the resistance box is used alone, one terminal of the box is
connected to both the C and c posts on the bridge and the other to
the T post.

The resistance of the individual steps of a decade will be
designated by means of subscripts, for example, the successive steps
of the 10 ohm decade will be designated as 10_1, 10_2, ---- 10_9, 10_x.
The sum of the steps which are included in any setting will be
designated without a subscript, for example, in the 10 ohm decade,
as 10, 20, ---- 90, X0.

Before beginning the bridge calibration the ratio arms should be
adjusted to equality. This adjustment of the ratio arms is necessary
because the 0.1 ohm decade is not in the same arm of the bridge as the
other decades. The bridge zero is not used in calibration; but, if it
is observed and found to have its usual value, there is added con-
fidence in the calibration. The current may be increased to obtain
greater sensitivity. If the external circuit is connected as indicated
above, the commutator must be set in the N (normal) position and not
in the R (reverse) position while calibrating.

The steps in the calibration are then as follows.

(a) Set 10 on the 10 ohm decade of the bridge and 0 on the 1 ohm
decade and use the lower decades to balance an appropriate resistance

in the external circuit. It is good practice to interpolate with galvanometer deflections to one tenth of a step on the lowest decade.

(b) Without disturbing the external circuit, change the bridge setting to 0 on the 10 ohm decade and X. on the 1 ohm decade and again use the lower decades to balance. It is convenient to adjust the slide wire in the external circuit so that the lower decade balances are near zero for the lesser of (a) or (b).

(c) Repeat (b) and then (a).

(d) Set 20 on the 10 ohm decade of the bridge and 0 on the 1 ohm decade and balance with a corresponding resistance in the external circuit, as in (a).

(e) Change the bridge setting to 10 in the 10 ohm decade and X. in the 1 ohm decade and proceed as in (b) and (c).

(f) Continue this process until each of the 10 ohm steps has been compared with the sum of the ten 1 ohm steps. The observations under (a) (b) and (c) yield the relation $10_1 + a_1 = X + b_1$ where a_1 and b_1 are the corrected averages of the respective lower decade readings, including galvanometer interpolations, which were obtained in the balancings of the bridge.

Similar equations are obtained from (d) and (e) leading to the complete set.

$$10_1 + a_1 = X. + b_1$$
$$10_2 + a_2 = X. + b_2$$

$$10_9 + a_9 = X. + b_9$$
$$10_x + a_x = X. + b_x$$

It is convenient to define the bridge unit by means of the relation,

$$X. = 10 \text{ bridge units} = 10 \text{ ohms (approximately)}.$$

The above relations will then yield the resistance of 10_1. 10_2, 10_3, etc. and by adding these successively the resistances of 10, 20, 30 etc. will be obtained.

For the 1 ohm decade, the procedure is similar and leads to similar equations. However, the resistance of O.X must be determined. To do this the equations are added to obtain their sum,

$$X. + \sum a = 10(O.X) + \sum b, \text{ so that}$$

$$O.X = \frac{X. + \sum (a - b)}{10}.$$

This value for the resistance of O.X is then substituted in the original equations and the rest of the procedure is the same as for the 10 ohm decade.

The calibration of the 0.1 ohm decade proceeds similarly.

So far the procedure has yielded calibration data for the 10, 1 and 0.1 ohm decades, and also the resistance of O.OX of the 0.01 ohm decade. A comparison of the value obtained for the resistance of O.OX with that of a previous calibration will indicate whether recalibration of the lower decades is advisable.

Calibration of the 0.01 ohm and lower decades is done in a similar manner by using the slide wire alone. The .0001 ohm steps, which ordinarily are the lowest, may be calibrated by comparing galvanometer deflections for each successive step.

If it is desired to calibrate the bridge in terms of absolute ohms or other units represented by a calibrated resistance standard,

the resistance standard, preferably one of 10 ohms, may be connected
to the bridge, just as a four lead thermometer, and measured in terms
of the X. of the 1 ohm decade. The result of this measurement
(bridge zero to be taken into account) leads to a relation between
the "bridge unit" and the unit of the standard. The corrections
previously obtained may be modified so that resistances are expressed
in terms of the unit represented by the standard.

The use of a reliable resistance standard has the advantage of
holding the bridge unit as constant as the resistance standard and
thus it may enable one to determine whether apparent drifts in R_O are
due to the thermometer or to the bridge. Another advantage in using
a resistance standard in calibrating bridges is that a thermometer may
be used interchangeably on these bridges with the precision to which
the bridge units agree. For work of the highest precision, however,
it is considered better practice, when using any thermometer, to
determine R_O at the time measurements are made and to be sure that the
bridge is self-consistent. The art of standard platinum resistance
thermometer construction and stabilization has now developed to
such an extent that the resistance of a well-treated thermometer
at a reliable fixed point may have a constancy comparable with that
of resistance standards.

APPENDIX A

Fixed Points and Reference Table for the
International Practical Temperature Scale of 1968

Defining fixed points of the IPTS-68*

Equilibrium state	Assigned value of International Practical Temperature	
	T	t
Equilibrium between the solid, liquid and vapour phases of equilibrium hydrogen (triple point of equilibrium hydrogen)	13.81 K	-259.34°C
Equilibrium between the liquid and vapour phases of equilibrium hydrogen at a pressure of 33 330.6 Nm⁻² (25/76 standard atmosphere)	17.042 K	-256.108°C
Equilibrium between the liquid and vapour phases of equilibrium hydrogen (boiling point of equilibrium hydrogen)	20.28 K	-252.87°C
Equilibrium between the liquid and vapour phases of neon (boiling point of neon)	27.102 K	-246.048°C
Equilibrium between solid, liquid and vapour phases of oxygen (triple point of oxygen)	54.361 K	-218.789°C
Equilibrium between the liquid and vapour phases of oxygen (boiling point of oxygen)	90.188 K	-182.962°C
Equilibrium between the solid, liquid and vapour phases of water (triple point of water)***	273.16 K	0.01°C
Equilibrium between the liquid and vapour phases of water (boiling point of water)** ***	373.15 K	100°C
Equilibrium between the solid and liquid phases of zinc (freezing point of zinc)	692.73 K	419.58°C
Equilibrium between the solid and liquid phases of silver (freezing point of silver)	1235.08 K	961.93°C
Equilibrium between the solid and liquid phases of gold (freezing point of gold)	1337.58 K	1064.43°C

*Except for the triple points and one equilibrium hydrogen point (17.042 K) the assigned values of temperature are for equilibrium states at a pressure of one standard atmosphere (p_0 = 101 325 Nm⁻²).

In the realization of the fixed points small departures from the assigned temperatures will occur as a result of failure to realise the required pressure exactly. If due allowance is made for these small temperature differences, they will not affect the accuracy of realization of the scale.

**The equilibrium state between the solid and liquid phases of tin (freezing point of tin) has the assigned value of t_{68} = 231.9681°C and may be used as an alternative to the boiling point of water.

***The water used should have the isotopic composition of ocean water.

Appendix A (continued)

The CCT-68 table giving W_t^* is defined by the expression,

$$t = \sum_{i=1}^{20} A_i (\ln W_t^*)^i ,$$

where the values of the coefficients A_i are

i	A_i	i	A_i
1	$0.2508462096788033 \times 10^3$	11	$0.7679763581708458 \times 10$
2	$0.1350998699649997 \times 10^3$	12	$0.2136894593828500 \times 10$
3	$0.5278567590085172 \times 10^2$	13	0.4598433489280693
4	$0.2767685488541052 \times 10^2$	14	$0.7636146292316480 \times 10^{-1}$
5	$0.3910532053766837 \times 10^2$	15	$0.9693286203731213 \times 10^{-2}$
6	$0.6556132305780693 \times 10^2$	16	$0.9230691540070075 \times 10^{-3}$
7	$0.8080358685598667 \times 10^2$	17	$0.6381165909526538 \times 10^{-4}$
8	$0.7052421182340520 \times 10^2$	18	$0.3022932378746192 \times 10^{-5}$
9	$0.4478475896389657 \times 10^2$	19	$0.8775513913037602 \times 10^{-7}$
10	$0.2125256535560578 \times 10^2$	20	$0.1177026131254774 \times 10^{-8}$

The function $W_t^*(t)$ is continuous at $t=0$ in its first and second derivatives with the function $W(t)$ given by equations (1), (2), and (3) for

$$\alpha = .0039259668 \text{ and } \delta = 1.496334 .$$

The complete authorized text of the International Practical Temperature Scale of 1968 appears in Metrologia 5, 35 (1969).

DIFFERENCES BETWEEN THE NUMERICAL VALUES OF TEMPERATURE GIVEN BY THE IPTS 1968 AND THE IPTS 1948 ($t_{68}-t_{48}$)